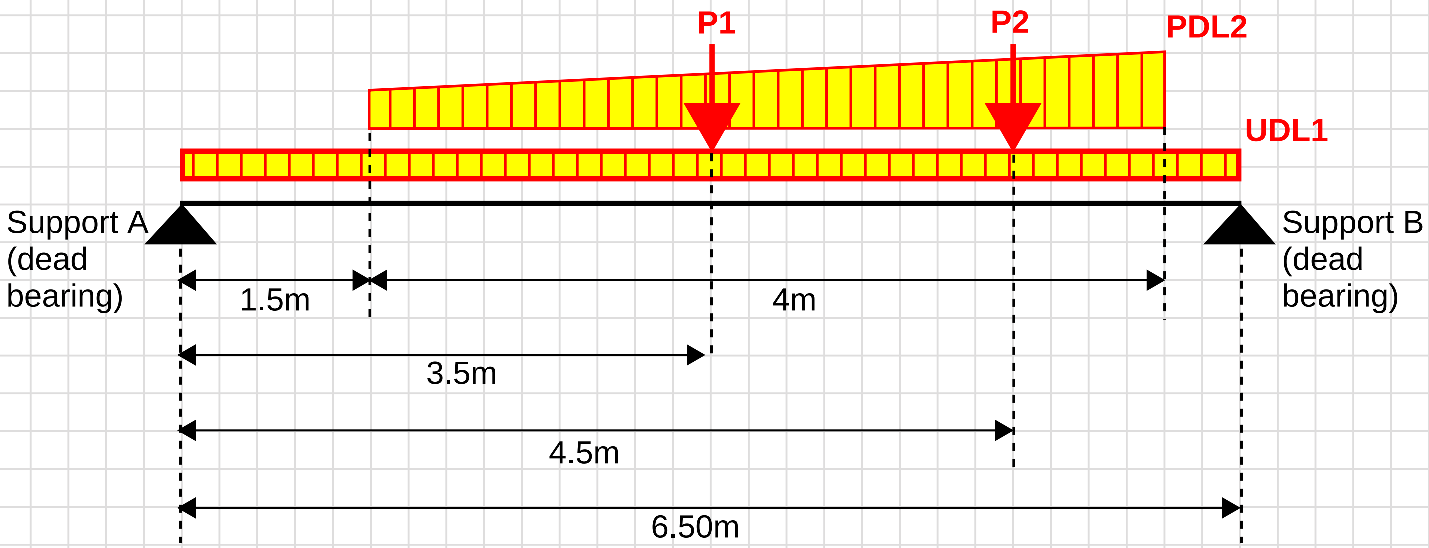
Design Spreadsheet: Steel Beam Analysis and Design	Date: 26/06/25
Subtitle: Steel Beam Analysis and Design	By: AL
	Version: 1

Applied Loading on Simply Supported Single Span Beam



UDL1 - Uniformly Distributed Load
Appllied Permanent "Dead" Load = 10kN/m (SLS unfactored load)
Appllied Imposed "Live" Load = 6.5kN/m (SLS unfactored load)

UDL2 - Partial Uniformly Distributed Load (with Varying Magnitude)
Appllied Permanent "Dead" Load = 5kN/m ---> 15kN/m (SLS unfactored load)
Appllied Imposed "Live" Load = 3kN/m --> 9kN/m (SLS unfactored load)

P1 - Point Load @ 3.5m from Left Support
Appllied Permanent "Dead" Load = 7kN (SLS unfactored load)
Appllied Imposed "Live" Load = 2.5kN (SLS unfactored load)

P2 - Point Load @ 4.5m from Left Support
Appllied Permanent "Dead" Load = 4kN (SLS unfactored load)
Appllied Imposed "Live" Load = 8kN (SLS unfactored load)

Chosen Beam Size
Universal Beam UB457x191x98
Mass of Beam M = 98.3kg/m (0.964kN/m SLS unfactored load)

Maximum Internal ULS Forces
Maximum Applied Bending Moment @ ULS M_{Ed} = 268kNm
Maximum Applied Shear Force @ ULS V_{Ed} = 152kN

Note:
SLS = Serviceability Limit State (i.e. unfactored loads used)
*ULS = Ultimate Limit State (i.e. factored loads used 1.35*Dead + 1.5*Live)*
For ULS load combination factors refer to Eurocode 0 i.e. BS EN1990:2002+A1:2005 and it's associated National Annex

	Design Spreadsheet: Steel Beam Analysis and Design	Date: 26/06/25
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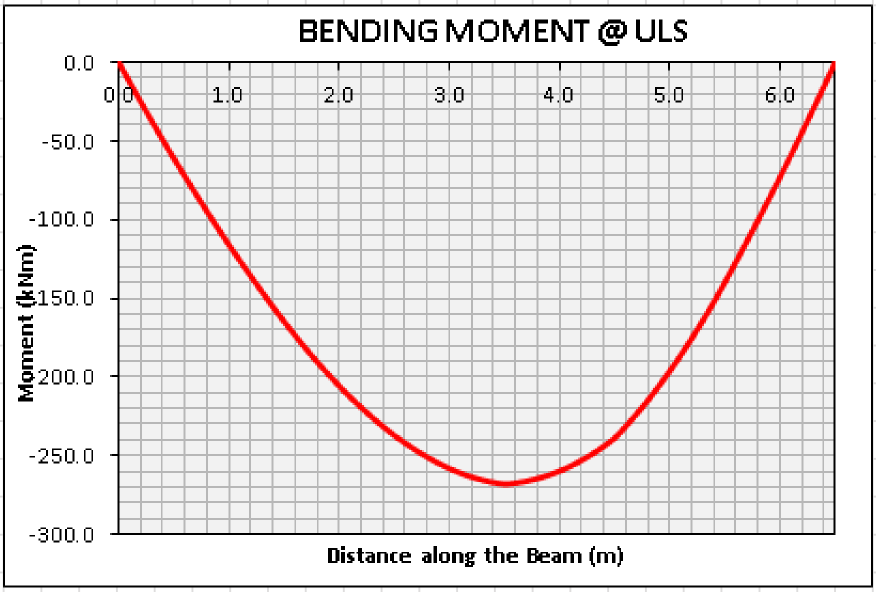
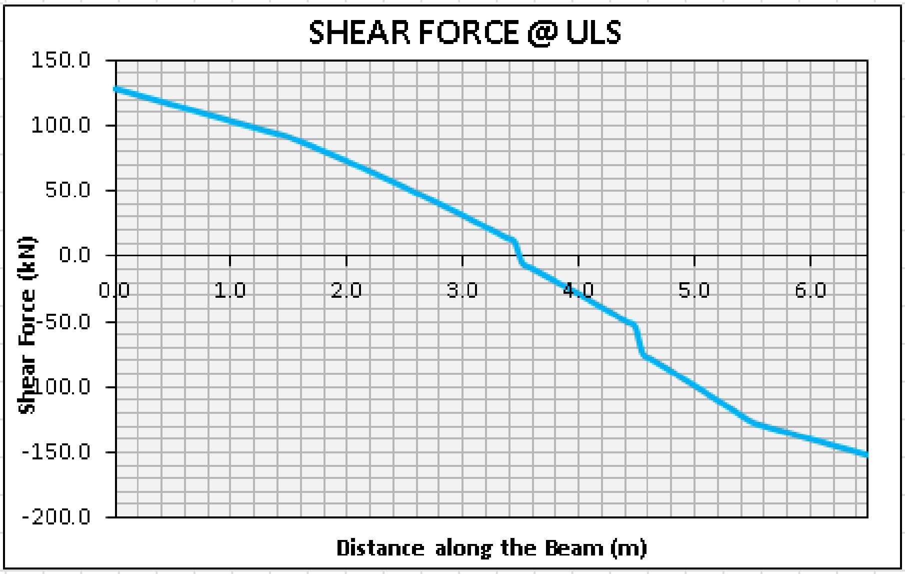
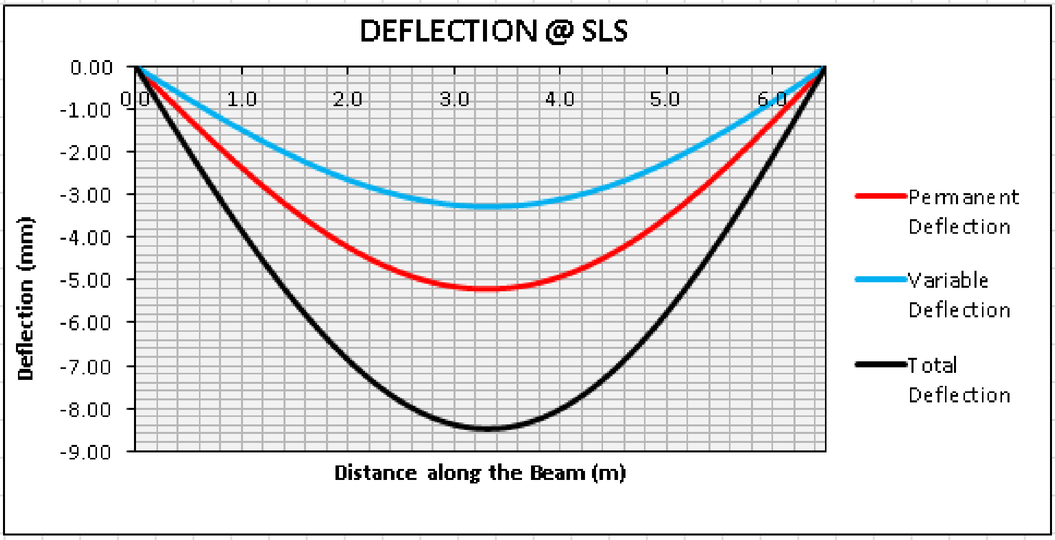
Bending Moment Diagram (from Spreadsheet)



Shear Force Diagram (from Spreadsheet)



Deflection Diagram (from Spreadsheet)



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Cross Sectional Properties of Chosen Steel Beam

Universal Beam UB457x191x98
Chosen Steel Grade: S355 (EN 10025-2)

Mass of Beam M = 98.3kg/m

Cross Sectional Area A = 12500mm^2

Depth of Section h = 467.2mm
Width of Section b = 192.8mm
Thickness of Web t_w = 11.4mm
Thickness of Flanges t_f = 19.6mm
Root Radius r = 10.2mm
Depth Between Fillets = 407.6mm

Second Moment of Area (Major Axis) I_y = 457,000,000mm^4
Second Moment of Area (Minor Axis) I_z = 23,500,000mm^4

Radius of Gyration (Major Axis) i_y = 191mm
Radius of Gyration (Minor Axis) i_z = 43.3mm

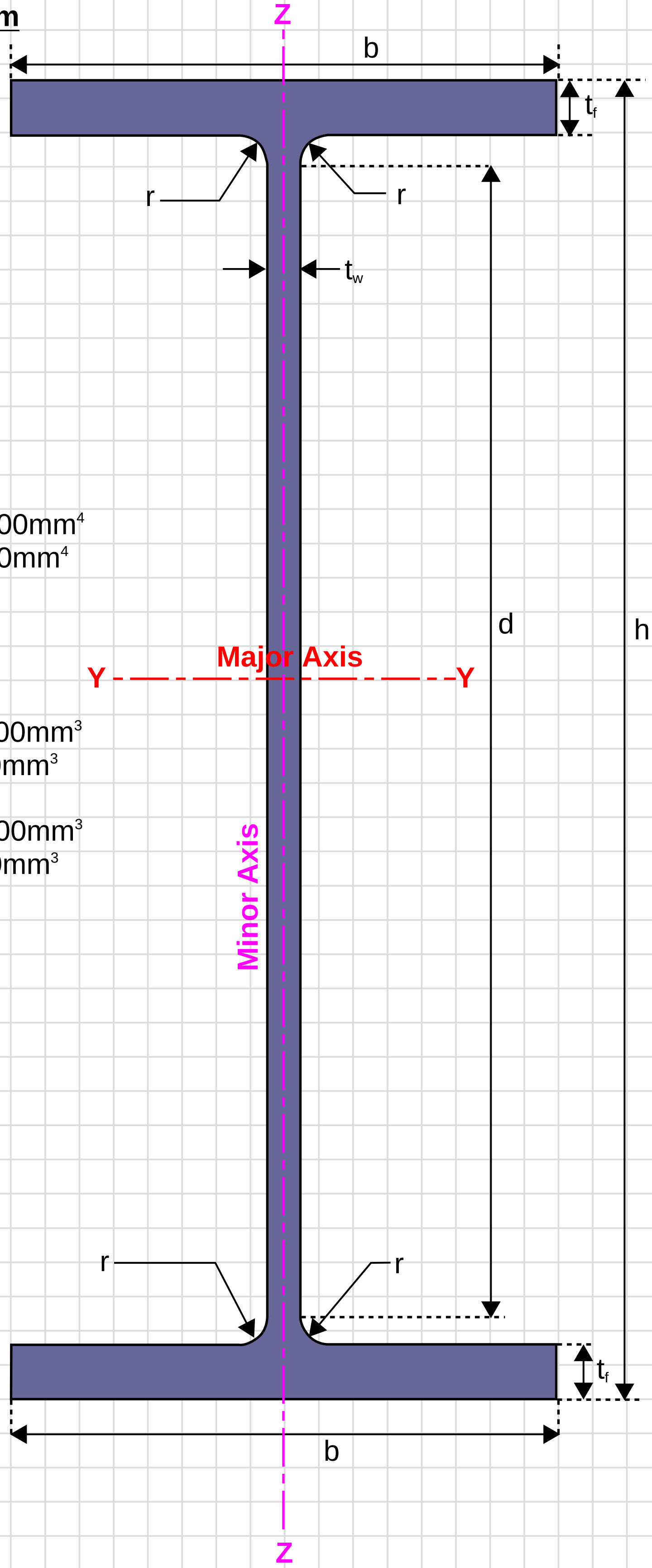
Elastic Section Modulus (Major Axis) $W_{el,y}$ = 1,960,000mm^3
Elastic Section Modulus (Minor Axis) $W_{el,z}$ = 243,000mm^3

Plastic Section Modulus (Major Axis) $W_{pl,y}$ = 2,230,000mm^3
Plastic Section Modulus (Minor Axis) $W_{pl,z}$ = 379,000mm^3

Buckling Parameter U = 0.881
Torsional Index x = 25.8

Warping Constant I_w = 1,180,000,000,000mm^6
Torsional Constant I_t = 1,210,000mm^4

Note:
Beam properties taken from Blue Book Online
https://www.steelforlifebluebook.co.uk/



Material Properties of Chosen Beam
Universal Beam UB457x191x98
Chosen Steel Grade: S355 (EN 10025-2)

Modulus of Elasticity E = 210,000N/mm^2

Poissons Ratio v = 0.3

Shear Modulus G = E / 2(1 + v)
Shear Modulus G = 210,000N/mm^2 / 2(1 + 0.3)
Shear Modulus G = 80769N/mm^2

BS EN1993-1-1 2005 - Section 3.2.6

Maximum thickness of the beam section t_{max} = max(t_f , t_w)
Maximum thickness of the beam section t_{max} = max(19.6mm , 11.4mm)
Maximum thickness of the beam section t_{max} = 19.6mm

Less than 40mm limit see table below

BS EN1993-1-1 2005 - Table 3.1

Standard and steel grade	Nominal thickness of the element t [mm]			
	$t \leq 40$ mm		40 mm $< t \leq 80$ mm	
	f_y [N/mm^2]	f_u [N/mm^2]	f_y [N/mm^2]	f_u [N/mm^2]
EN 10025-2				
S 235	235	360	215	360
S 275	275	430	255	410
S 355	355	AC2 490 AC2	335	470
S 450	440	550	410	550
EN 10025-3				
S 275 N/NL	275	390	255	370
S 355 N/NL	355	490	335	470
S 420 N/NL	420	520	390	520
S 460 N/NL	460	540	430	540
EN 10025-4				
S 275 M/ML	275	370	255	360
S 355 M/ML	355	470	335	450
S 420 M/ML	420	520	390	500
S 460 M/ML	460	540	430	530
EN 10025-5				
S 235 W	235	360	215	340
S 355 W	355	AC2 490 AC2	335	490
EN 10025-6				
S 460 Q/QL/QL1	460	570	440	550

Yield Strength of Steel f_y = 355N/mm^2
Ultimate Tensile Strength of Steel f_u = 490N/mm^2

Determine Section Class

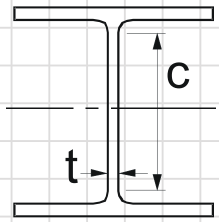
$\varepsilon = \sqrt{[235 / f_y]}$
$\varepsilon = \sqrt{[235 / 355N/mm^2]}$
$\varepsilon = 0.814$

BS EN1993-1-1 2005 - Tables 5.1 & 5.2

Section Class of Beam Web

The part of the beam we are considering is the web. The web of the beam is subject to bending **only**

Calculate ratio of element length to element thickness (c / t)
The element length (c) is the depth between fillets (d)
The element thickness (t) is the thickness of the beam web (t_w)



$c / t = d / t_w$
$c / t = 407.6mm / 11.4mm$
$c / t = 35.75$

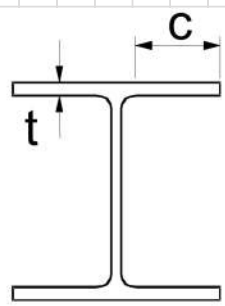
c / t ≤ 72ε	c / t ≤ 83ε	c / t ≤ 124ε
35.75 ≤ 58.6	35.75 ≤ 67.6	35.75 ≤ 100.936
Class 1	Class 2	Class 3

In this example the web of the beam is class 1
BS EN1993-1-1 2005 - Table 5.2 (sheet 1 of 3)

Section Class of Beam Flanges

The part of the beam we are considering is the flanges. The flanges of the beam are subject to compression **only**

Calculate ratio of element length to element thickness (c / t)
The element length (c) is the outstand length of the flange
The element thickness (t) is the thickness of the beam flanges (t_f)



$c / t = (0.5b - 0.5t_w - r) / t_f$
$c / t = (0.5*192.8mm - 0.5*11.4mm - 10.2mm) / 19.6mm$
$c / t = 4.107$

c / t ≤ 9ε	c / t ≤ 10ε	c / t ≤ 14ε
4.107 ≤ 7.326	4.107 ≤ 8.140	4.107 ≤ 11.396
Class 1	Class 2	Class 3

In this example the flanges of the beam are class 1
BS EN1993-1-1 2005 - Table 5.2 (sheet 2 of 3)

Section Class Continued

Section Class of Beam Overall = max(Section Class of Beam Web, Section Class of Beam Flanges)
Section Class of Beam Overall = max(1, 1)
Section Class of Beam Overall = 1

Partial Material Safety Factors

Resistance of the cross section whatever the class is (γ_{M0}) = 1.0

Resistance of the members to instability assessed by member checks (γ_{M1}) = 1.0

Resistance of cross sections in tension to fracture (γ_{M2}) = 1.10

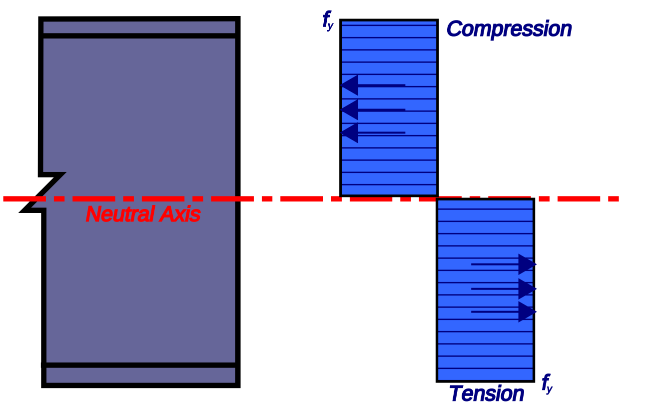
BS EN1993-1-1 2005 - Section 6.1 NOTE 2B & UK National Annex Section NA.2.15

Calculate Bending Moment Capacity (No Allowance for Buckling at this Stage)

Bending Moment Resistance for Class 1 or Class 2 Beams (Plastic Capacity)

$M_{c,Rd} = W_{pl,y} * f_y * (1 / \gamma_{M0})$
$M_{c,Rd}$ = 2,230,000mm^3 * 355N/mm^2 * (1 / 1.0)
$M_{c,Rd}$ = 792kNm

Beam on Elevation | Bending Stress Profile

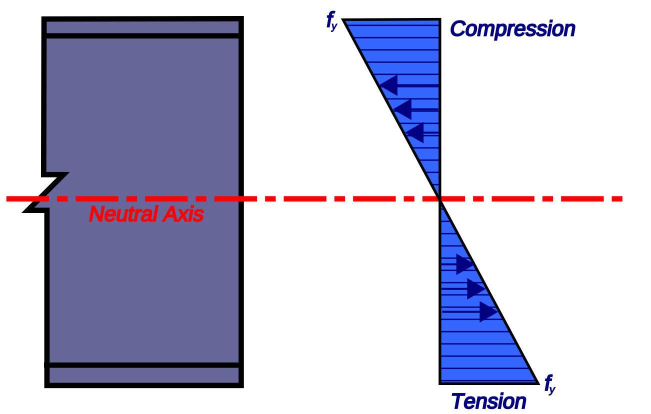


BS EN1993-1-1 2005
Section 6.2.5(2)
Equation 6.13

Bending Moment Resistance for Class 3 Beams (Elastic Capacity)

$M_{c,Rd} = W_{el,y} * f_y * (1 / \gamma_{M0})$
$M_{c,Rd}$ = 1,960,000mm^3 * 355N/mm^2 * (1 / 1.0)
$M_{c,Rd}$ = 696kNm

Beam on Elevation | Bending Stress Profile



BS EN1993-1-1 2005
Section 6.2.5(2)
Equation 6.14

In this example the beam is class 1 so we can use the plastic moment capacity of the beam
$M_{c,Rd}$ = 792kNm

Utilisation of the beam in bending
= applied ULS moment / moment capacity
= M_{Ed} / $M_{c,Rd}$
= 268kNm / 792kNm
= 34% --> OK

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<u>Calculate Shear Capacity of the Beam</u>

Height of the beam web h_w = h - tf - tf
Height of the beam web h_w = 467.2mm - 19.6mm - 19.6mm
Height of the beam web h_w = 428mm

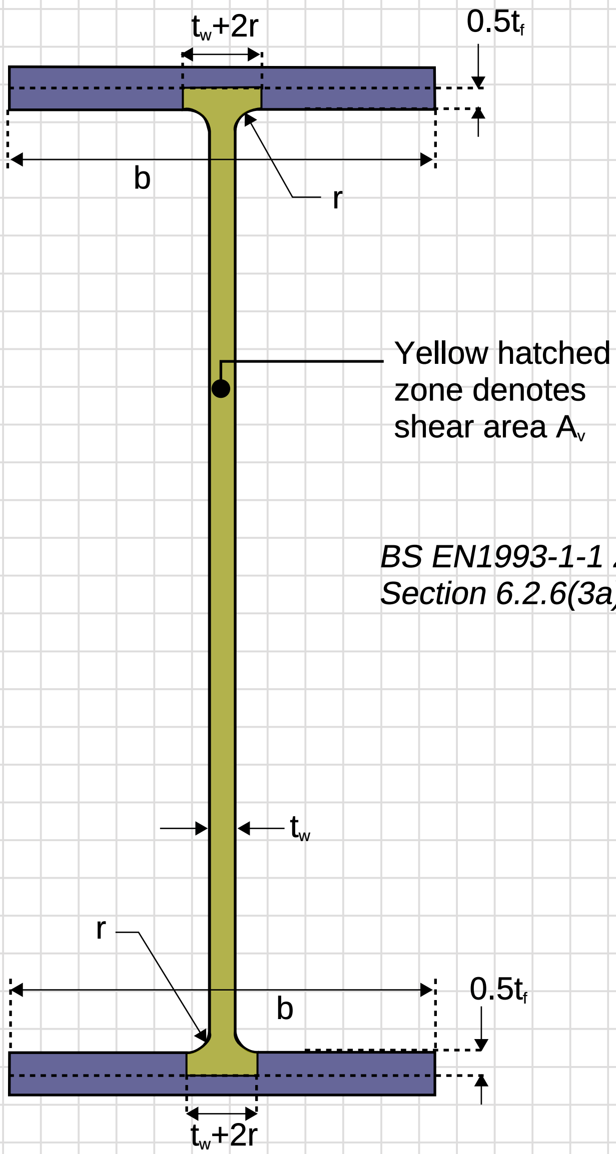
Shear buckling factor η = 1.0 (BS EN1993-1-1 & 1-5 Recommended Value)

Does Shear Buckling of the web need to be considered?
$h_w / t_w > 72\varepsilon / \eta$
428mm / 19.6mm > (72 * 0.814) / 1.0
21.836 > 58.608
No as 21.836 is less than 58.608, shear buckling of the web can be ignored

if you need to account for shear buckling you need to go to BS EN1993-1-5 which is the steel code for design of flat plated elements

Shear Area of the Beam $A_v = \max\{ A - (2 * b * t_f) + [(t_w + 2*r) * t_f] \;,\; \eta * h_w * t_w\}$

$A_v = \max\{ 12500\text{mm}^2 - (2 * 192.8\text{mm} * 19.6\text{mm}) + [(11.4\text{mm} + 2*10.22\text{mm}) * 19.6\text{mm}] \;,\; 1.0 * 428\text{mm} * 11.4\text{mm}\}$
$A_v = \max\{ 5566.3\text{mm}^2 \;,\; 4879.2\text{mm}^2\}$
$A_v = 5566.3\text{mm}^2$



BS EN1993-1-1 2005
Section 6.2.6(3a)

Plastic Shear Capacity $V_{pl,Rd} = A_v * f_y * (1 / \sqrt{3.0}) * (1 / \gamma_{M0})$
Plastic Shear Capacity $V_{pl,Rd} = 5566.3\text{mm}^2 * 355\text{N/mm}^2 * (1 / \sqrt{3.0}) * (1 / 1.0)$
Plastic Shear Capacity $V_{pl,Rd}$ = 1141kN

BS EN1993-1-1 2005
Section 6.2.6(2)
Equation 6.18

Calculate Shear Capacity of the Beam (Continued)

Utilisation of the beam in shear
= applied ULS shear / plastic shear capacity
= V_{Ed} / $V_{pl,Rd}$
= 152kN / 1141kN
= 13% --> OK

Check for Combined Bending Moment & Shear Force

If the applied maximum ULS shear force is greater than or equal to 50% of the plastic shear capacity of the beam then you **must** apply a reduction to the bending moment capacity of the beam to account for combined shear force and bending moment. (As outlined in BS EN1993-1-1 Section 6.2.8)

Applied ULS shear force ≥ 50% plastic shear capacity of the beam
$V_{Ed} \geq 0.5V_{pl,Rd}$
152kN ≥ 0.5 * 1141kN
152kN ≥ 571kN --> NO

As can be seen by the inequality above, the applied ULS shear force in this example does not exceed the 50% plastic shear capacity of the beam. Therefore no reduction is required on the moment capacity for co-existing shear force.

However, for completeness of this example, the methodology to reduce the bending moment capacity due to co-existing shear force is outlined below.

In BS EN1993-1-1 Section 6.2.8(3) a reduction factor is provided. This reduction factor is to be applied to the yield strength of the steel (f_y) to reduce the bending moment capacity of the beam

Reduction Factor $$\rho = \left(\frac{2V_{Ed}}{V_{pl,Rd}} - 1\right)^2$$

Reduced yield strength of the beam = $(1 - \rho) * f_y$

You could apply this reduction to the steel yield strength for the entire beam using the following equation below to work out the reduced bending moment capacity:

$M_{c,Rd} = (1 - \rho) * f_y * W_{pl,y} * (1 / \gamma_{M0})$ - for class 1 & 2 sections
$M_{c,Rd} = (1 - \rho) * f_y * W_{el,y} * (1 / \gamma_{M0})$ - for class 3 sections

However when you do this you massively under estimate the capacity of the beam.
The reason for this is as follows:

- The web of the I-beam carries the majority of the shear force and not much of the bending
- The flanges of the I-beam carries the majority of the bending moment and not much of the shear force.

Therefore it's more appropriate to apply the reduction for shear force to the portion of the beam which is carrying the bulk of the shear force which is the web of the beam. The following equations do this to get better results on the reduced moment capacity of the beam.

Check for Combined Bending Moment & Shear Force (Continued)

Plastic Moment capacity of the beam accounting for the co-existing shear force on the web only

$$M_{y,V,rd} = \frac{\left[\overbrace{W_{pl,y} - \frac{\rho A_w^2}{4t_w}}^{\text{reduced plastic section modulus for co-existing shear}}\right] f_y}{\gamma_{M0}}$$

BS EN1993-1-1 Equation 6.30 For class 1 & 2 sections using the plastic section modulus

Elastic Moment capacity of the beam accounting for the co-existing shear force on the web only

$$M_{y,V,rd} = \frac{\left[\overbrace{W_{el,y} - \frac{\rho A_w^2}{6t_w}}^{\text{reduced elastic section modulus for co-existing shear}}\right] f_y}{\gamma_{M0}}$$

BS EN1993-1-1 Equation 6.30 (modified) For class 3 sections using the elastic section modulus

Please note: The moment capacities shown above cannot exceed the basic bending moment capacity that was calculated previously. If you have run the calculations above and got a higher bending moment capacity than before you have done something wrong

The equations shown above are poorly explained in the Eurocode, so I've provided a step-by-step explanation below. Firstly lets outline what we are trying to achieve.

1.) We want to account for the co-existing applied shear force with the applied bending moment

2.) We want to account for this co-existing shear force by applying a reduction factor p to the moment capacity of the beam

3.) We want the reduction factor for shear to **only** apply to the bending capacity of the **web of the beam** (because it's the web of the beam which **predominately** carries the shear force)

4.) To do this we need to separate out the bending capacity of the **beam web** in order to apply our reduction factor p. We can then subtract this from the overall moment capacity of the beam.

Plastic section modulus for the entire beam (mm^3)

Plastic section modulus for the web of the beam with the shear reduction applied (mm^3)

$$M_{y,V,rd} = \frac{\left[W_{pl,y} - \frac{\rho A_w^2}{4t_w}\right] f_y}{\gamma_{M0}}$$

Yield strength of the beam (N/mm^2)

Plastic Moment capacity of the beam accounting for the co-existing shear force on the web only (kNm)

Partial material safety factor (= 1.0)

Check for Combined Bending Moment & Shear Force (Continued)

$$M_{y,V,rd} = \frac{\left[W_{el,y} - \frac{\rho A_w^2}{6t_w}\right] f_y}{\gamma_{M0}}$$

Elastic section modulus for the entire beam (mm^3)

Elastic section modulus for the web of the beam with the shear reduction applied (mm^3)

Yield strength of the beam (N/mm^2)

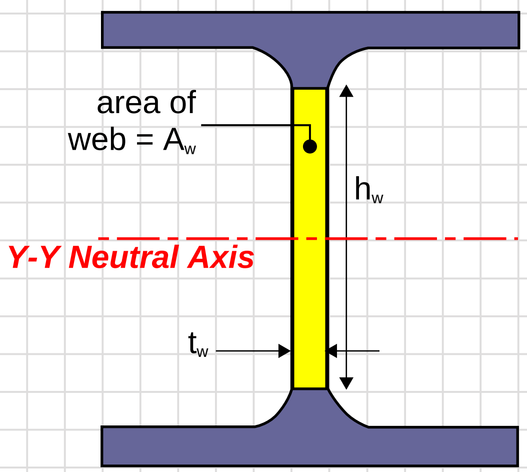
Elastic Moment capacity of the beam accounting for the co-existing shear force on the web only (kNm)

Partial material safety factor (= 1.0)

The portion of the equations of most interest is the plastic/elastic section modulus for the web of the beam with the shear reduction applied (highlighted in yellow above). This is where we work out how the web of the beam contributes to the bending moment capacity and where we apply the reduction factor for co-existing shear.

The web of the beam is rationalised as a rectangle with width t_w and height h_w. Luckily the plastic and elastic section modulus for a rectangle is widely known and is equal to $bd^2/4$ and $bd^2/6$ for the plastic and elastic section modulus respectively (b = rectangle width, d = rectangle depth).

The diagram and formulas below illustrate how these formulas for the plastic and elastic section modulus for a rectangle are applied to our beam web.



Plastic section modulus of a rectangle = width * depth^2 * 1/4
Plastic section modulus of beam web = t_w * h_w^2 * 1/4
Substitute A_w = t_w * h_w
Plastic section modulus of beam web = A_w^2 / $4t_w$
Plastic section modulus of beam web with shear reduction
= ρ * A_w^2 / $4t_w$

Elastic section modulus of a rectangle = width * depth^2 * 1/6
Elastic section modulus of beam web = t_w * h_w^2 * 1/6
Substitute A_w = t_w * h_w
Elastic section modulus of beam web = A_w^2 / $6t_w$
Elastic section modulus of beam web with shear reduction
= ρ * A_w^2 / $6t_w$

So in summary the equations above are doing the following:
1.) Calculating the section modulus (plastic or elastic) for the web of the beam by modelling this as a rectangle with width t_w and height h_w
2.) Applying the shear reduction factor to the section modulus of the beam web
3.) subtracting this reduced section modulus for the beam web from the overall section modulus
4.) using this final section modulus to calculate the reduced moment capacity of the beam which now accounts for the co-existing shear.

Lateral Torsional Buckling (LTB)

Destabilising Load

Is the applied transverse loading destabilising? --> NO
Destabilising load factor D = 1.0

A destabilising load is one which, when applied to the beam, contributes to the lateral torsional buckling of the beam. This is typically a load which has the following features:

1.) The load is applied above the shear centre of the beam
(for an I-beam the shear centre coincides with the geometric centroid)

2.) The load can move with the beam when the beam starts to buckle laterally.

A good example of a destabilising load is a free standing masonry wall supported on the top flange of an I-beam. As the beam starts to buckle to the side via the LTB failure mode the wall will move with the beam. The weight of the wall will continue to drive the lateral buckling of the beam essentially driving (or exacerbating) the failure response of the beam.

Destabilising loads are accounted for via a D factor which is applied directly to the calculation for slenderness λ. A typical value of D for a destabilising load is D = 1.2 which is essentially a 20% increase on the slenderness of the beam. This is mentioned in SCI P360 page 30.

C_1 Factor

The C_1 factor accounts for the shape of the bending moment diagram in the lateral torsional buckling response.
Higher values of C_1 give better moment capacities so using C_1 = 1.0 is the conservative approach.

A formula to calculate C_1 based on the shape of the bending moment diagram is provided in Wong & Driver AISC Eng Journal, Q1 2010 Table 2 and is outlined below for this worked example.

$C_1 = \min\{ 2.5 , 4M_{Ed} / \sqrt{[M_{Ed}^2 + 4M_a^2 + 7M_b^2 + 4M_c^2]}\}$ *Wong & Driver AISC Eng Journal, Q1 2010 Table 2*

M_{Ed} = maximum applied ULS bending moment
M_a = applied ULS bending moment at quarter point on the beam (i.e. @ 0.25L)
M_b = applied ULS bending moment at midspan of the beam (i.e. @ 0.5L)
M_c = applied ULS bending moment at 3/4 point on the beam (i.e. @ 0.75L)

$C_1 = \min\{ 2.5 , 4*268kNm / \sqrt{[(268kNm)^2 + 4(176kNm)^2 + 7(265kNm)^2 + 4(208kNm)^2]}\}$
$C_1 = \min\{ 2.5 , 1.156\}$
$C_1 = 1.156$

Lateral Torsional Buckling (LTB) (Continued)

Effective Length of the Beam

The effective length of the beam is used to account for the amount of restraint available at the ends of the beam (i.e. the support conditions). The effective length is calculated for each support condition at each end of the beam and then an average effective length is taken for the beam as a whole.

The effective length is obtained from the previous British Standard steel code BS 5950-1:2000 Table 13 which is replicated below.

In this worked example we have a normal non-destabilising load and each end of the beam is supported on a dead bearing i.e. the beam is just resting on the support. In real life this would be like supporting a beam on a wall or a concrete padstone with no positive fixings to hold it down. In the table below this relates to *"Partial torsional restraint against rotation about longitudinal axis provided only by pressure of bottom flange onto supports"*

Table 13 - Effective Length L_{eff} for beams without intermediate restraint

Conditions of restraint at supports		Loading condition: Normal	Loading condition: Destabilising
Compression flange laterally restrained Nominal torsional restraint against rotation about longitudinal axis as given in 4.2.2	Both flanges fully restrained against rotation on plan	0.7L	0.85L
	Compression flange fully restrained against rotation on plan	0.75L	0.9L
	Both flanges partially restrained against rotation on plan	0.8L	0.95L
	Compression flange partially restrained against rotation on plan	0.85L	1.0L
	Both flanges free to rotation on plan	1.0L	1.2L
Compression flange laterally unrestrained Both flanges free to rotate on plan	Partial torsional restraint against rotation about longitudinal axis provided by connection of bottom flange to supports	1.0L + 2h	1.2L + 2h
	Partial torsional restraint against rotation about longitudinal axis provided only by pressure of bottom flange onto supports	1.2L + 2h	1.4L + 2h
h = height of the beam on cross section			

Effective length of beam due to Support A @ beginning of beam (dead bearing):
L_{eff1} = (1.2 * Beam Span) + (2 * Beam Height)
L_{eff1} = 1.2L + 2h
L_{eff1} = (1.2 * 6500mm) + (2 * 467.2mm)
L_{eff1} = 8734.4mm

Effective length of beam due to Support B @ end of beam (dead bearing):
L_{eff2} = (1.2 * Beam Span) + (2 * Beam Height)
L_{eff2} = 1.2L + 2h
L_{eff2} = (1.2 * 6500mm) + (2 * 467.2mm)
L_{eff2} = 8734.4mm

Effective length L_{eff} = (L_{eff1} + L_{eff2}) / 2 = (8734.4mm + 8734.4mm) / 2 = 8734.4mm
Effective length factor k = L / L_{eff} = 6500mm / 8734.4mm = 1.34

Lateral Torsional Buckling (LTB) (Continued)

g Factor to Allow for the In-plane Curvature of the Beam Prior to Buckling

This g factor is applied when calculating the elastic critical buckling moment for lateral torsional buckling (M_{cr}). This factor has a maximum value of 1.0 with lower values giving more beneficial results for the LTB capacity. Using g = 1.0 is the conservative approach.

$$g = \sqrt{1 - \frac{I_z}{I_y}}$$

$$g = \sqrt{1 - \frac{23,500,000mm^4}{457,000,000mm^4}}$$

g = 0.974

Steel Construction Institute (SCI) Publication P360 Section 2.2.1 Page 11

The general expression for the elastic critical buckling moment (M_{cr}) in its classic form doesn't consider how the beam is bending in-plane prior to buckling. That is, it assumes a straight member under constant moment, which isn't always realistic.

The g factor modifies the elastic critical buckling moment to reflect in-plane curvature. Specifically, it is used in cases where the beam is continuously bent (as in a moment gradient, like in a simply supported beam under point load at midspan).

When a beam is already curving in-plane (i.e. bending about it's strong Y-Y axis), the lateral stiffness of the beam is enhanced due to the "restoring" effect of this in-plane curvature. This makes it more stable, and thus increases the elastic critical buckling moment (M_{cr})

In-plane curvature acts like a prestressing effect in the direction of loading — when the beam is already in a curved configuration, small lateral displacements are resisted more strongly.

From an energy perspective, the system needs more energy to buckle laterally because some of the energy is already stored in the in-plane bending.

This effect is most pronounced in beams with moment gradients (non uniform moment), such as in real-world scenarios where loading is applied at discrete points (i.e. point loads on the beam).

Lateral Torsional Buckling (LTB) (Continued)

Elastic Critical Buckling Moment

The elastic critical buckling moment for lateral torsional buckling (M_{cr}) can be thought of as an upper bound on the buckling capacity of the beam. If you could fabricate a perfectly straight and perfectly horizontal beam and then apply a perfectly plumb and centralised transverse load on this beam and then have perfect support conditions then the buckling capacity of the beam would equal M_{cr}.

Unfortunately we live in the real world and we can **never** achieve this level of perfection. This is why M_{cr} is an upper bound on the lateral torsional buckling capacity. It is an upper bound we will never achieve in reality.

To deal with this we first calculate M_{cr} (the theoretical upper bound) and then apply reductions to this value using the buckling curves (these curves are also known as the Perry–Robertson buckling curves). Once we have applied reductions we obtain the 'actual' lateral torsional buckling capacity ($M_{b,Rd}$) which we can compare to our applied ULS bending moment (M_{Ed}) to see if our beam passes.

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{g(kL)^2} \left(\sqrt{\left(\frac{k}{k_w}\right)^2 \frac{I_w}{I_z} + \frac{(kL)^2 G I_t}{\pi^2 E I_z} + (C_2 z_g)^2} - C_2 z_g \right)$$

Steel Construction Institute (SCI) Publication P360 Section 2.2.1 Equation 2.14 Page 11

C_1 = factor to account for bending moment shape (calculated previously)

E = Modulus of elasticity (defined previously) (N/mm^2)

I_z = second moment of area about minor Z-Z axis (defined previously) (mm^4)

g = factor to account for in-plane curvature of the beam prior to buckling (calculated previously)

k = effective length factor (defined previously)

L = beam length (defined previously) (mm)

kL = effective length of the beam (also denoted as L_{eff}) (defined previously) (mm)

k_w = Warping restraint parameter
Where no warping restraint is provided and as a conservative assumption when the degree of warping restraint is uncertain k_w should be taken as unity (1.0). For this example and the Excel spreadsheet k_w is taken as 1.0 and the equation above simplified.

I_w = warping constant (defined previously) (mm^6)

G = shear modulus (defined previously) (N/mm^2)

I_t = torsion constant (defined previously) (mm^4)

C_2 = Parameter associated with the load level and is dependant on the shape of the bending moment diagram

z_g = Distance between the level of application of the loading and the shear centre. z_g is positive for destabilising for loads applied above the shear centre (i.e. loads applied to beam top flange)

Lateral Torsional Buckling (LTB) (Continued)

Elastic Critical Buckling Moment (Continued)

The general form of equation 2.14 shown on the previous page is simplified to the form shown below. This is done because:

1.) The C_2 factor is very difficult to determine apart from in very simple cases and we want a general formula.

2.) The warping restraint factor (k_w) can be conservatively taken as 1.0

3.) The z_g factor is to account for destabilising loads. However we are accounting for destabilising loads via a D factor which will be applied directly to the slenderness (λ) later in the calculation. So this can be taken as zero.

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{g(kL)^2}\left(\sqrt{\left(\frac{k}{k_w}\right)^2 \frac{I_w}{I_z} + \frac{(kL)^2 G I_t}{\pi^2 E I_z} + (C_2 z_g)^2} - C_2 z_g\right)$$

These disappear from the formula

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{g(kL)^2}\sqrt{\frac{I_w}{I_z} + \frac{(kL)^2 G I_t}{\pi^2 E I_z}}$$

Steel Construction Institute (SCI) Publication P360
Section 2.2.1 Equation 2.14 Page 11
(Simplified form)

$$M_{cr} = 1.156 \frac{\pi^2 * 210,000\frac{N}{mm^2} * 23,500,000mm^4}{0.974(1.34 * 6500mm)^2}\sqrt{\frac{1,180,000,000,000mm^6}{23,500,000mm^4} + \frac{(1.34 * 6500mm)^2 * 80769\frac{N}{mm^2} * 1,210,000mm^4}{\pi^2 * 210,000\frac{N}{mm^2} * 23,500,000mm^4}}$$

M_{cr} = 342.8kNm

Lateral Torsional Buckling (LTB) (Continued)

Slenderness

Slenderness is a ratio of the bending moment resistance of the beam with the elastic critical buckling moment i.e. slenderness is equal to:

$$\bar{\lambda}_{LT} = D\sqrt{\frac{\text{Moment capacity of the beam without LTB } (M_{c,Rd})}{\text{Elastic critical buckling moment } (M_{cr})}}$$

BS EN1993-1-1 section 6.3.2.2(1) with additional D factor for destabilising load as per SCI P

The larger the value of slenderness the more the beam wants to buckle, i.e. the lower the lateral torsional buckling resistance of the beam.

$$\bar{\lambda}_{LT} = D\sqrt{\frac{M_{c,Rd}}{M_{cr}}}$$

$$\bar{\lambda}_{LT} = 1.0 * \sqrt{\frac{792kNm}{342.8kNm}}$$

λ_{LT} = 1.52

BS EN1993-1-1 section 6.3.2.2(1) with additional factor D for the destabilising load as per Steel Construction Institute (SCI) publication P360 Section 2.5

Buckling Curve

h / b = 467.2mm / 192.8mm = 2.423

All universal beams and universal columns are rolled sections

Determine the buckling curve to use from modified Table 6.5 in UK National Annex to BS EN1993-1-1

BS EN 1993-1-1:2005, Table 6.5 should be replaced with the following table:

Cross-section	Limits	Buckling curve
Rolled doubly symmetric I and H sections and hot-finished hollow sections	h/b ⩽ 2	b
	$\boxed{2.0 < h/b \leqslant 3.1 \quad c}$	
	h/b > 3.1	d
Angles (for moments in the major principal plane)		d
All other hot-rolled sections.		d
Welded doubly symmetric sections and cold-formed hollow sections	h/b ⩽ 2	c
	2.0 ⩽ h/b < 3.1	d

Use buckling curve C

Imperfection Factor

Determine the imperfection factor to use from Table 6.3 in BS EN1993-1-1

Table 6.3: Recommended values for imperfection factors for lateral torsional buckling curves

Buckling curve	a	b	c	d
Imperfection factor α_{LT}	0,21	0,34	$\boxed{0,49}$	0,76

α_{LT} = 0.49

Lateral Torsional Buckling (LTB) (Continued)

Perry–Robertson Buckling Curves & Formulas

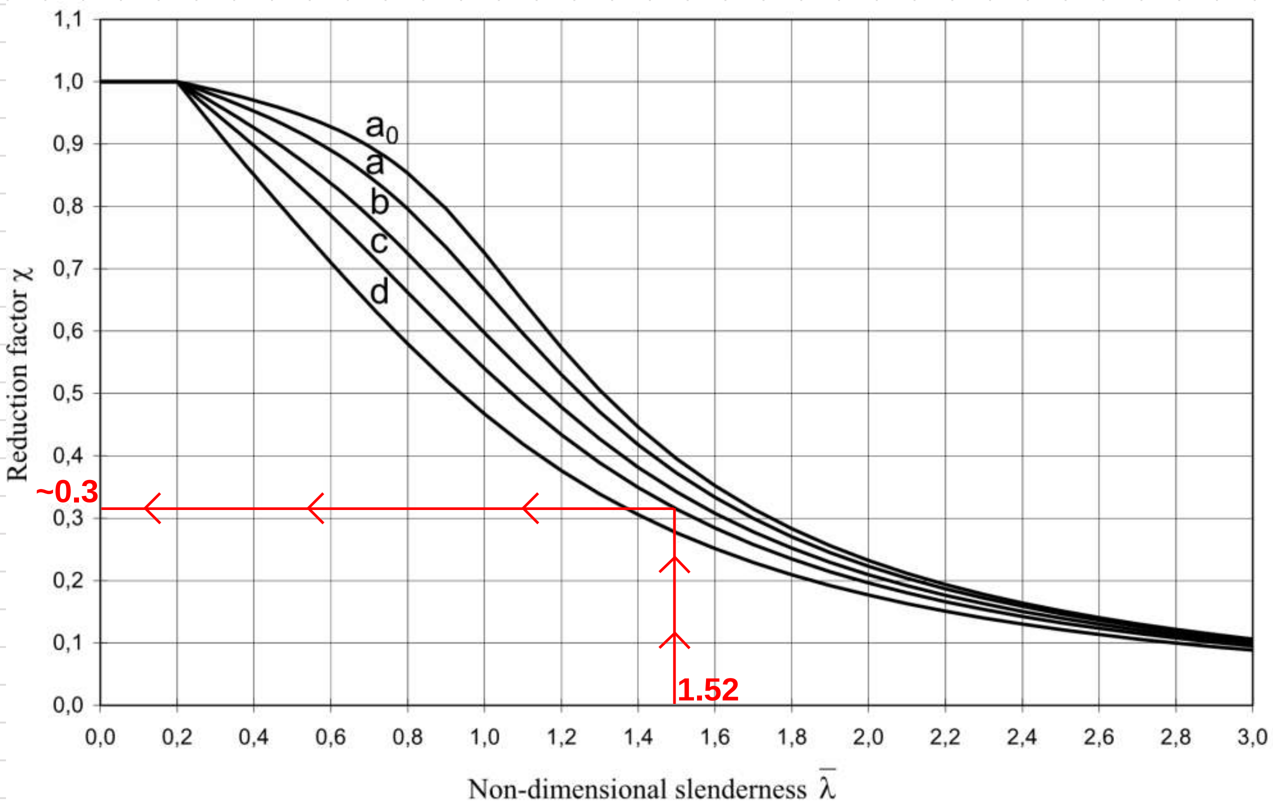


Figure 6.4: Buckling curves

Our reduction factor for LTB should be circa 0.3 as per the graph above from BS EN1993-1-1

$\bar{\lambda}_{LT,0}$ = 0.40 — *BS EN1993-1-1 Section 6.3.2.3(1) & NA.2.17*
(0.40 is recommended value for rolled sections)

β = 0.75 — *BS EN1993-1-1 Section 6.3.2.3(1) & NA.2.17*
(0.75 is recommended value for rolled sections)

$$\phi_{LT} = 0.5[1 + \alpha_{LT}(\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}) + \beta\bar{\lambda}_{LT}^2]$$

$$\phi_{LT} = 0.5[1 + 0.49(1.52 - 0.40) + 0.75 * 1.52^2]$$

ϕ_{LT} = 1.64

BS EN1993-1-1 Section 6.3.2.3(1)

$$\chi_{LT} = min(1.0, \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \beta\bar{\lambda}_{LT}^2}}, \frac{1}{\bar{\lambda}_{LT}^2})$$

$$\chi_{LT} = min(1.0, \frac{1}{1.64 + \sqrt{1.64^2 - 0.75 * 1.52^2}}, \frac{1}{1.52^2})$$

$$\chi_{LT} = min(1.0, 0.382, 0.433)$$

χ_{LT} = 0.382

Initial Reduction Factor for LTB

BS EN1993-1-1 Section 6.3.2.3(1) Equation 6.57

Lateral Torsional Buckling (LTB) (Continued)

Modified Perry–Robertson Buckling Curves & Formulas

$$k_c = \frac{1}{\sqrt{C_1}}$$

$$k_c = \frac{1}{\sqrt{1.156}}$$

$k_c = 0.930$

Slenderness correction factor

UK NA to BS EN1993-1-1 Section NA.2.18

$$f = min(1.0, 1 - 0.5(1 - k_c)[1 - 2(\bar{\lambda}_{LT} - 0.8)^2])$$

$$f = min(1.0, 1 - 0.5(1 - 0.93)[1 - 2(1.52 - 0.8)^2])$$

$$f = min(1.0, 1.0)$$

$$f = 1.0$$

Factor to Account for Moment Distribution Between the Lateral Restraints

BS EN1993-1-1 Section 6.3.2.3(2)

$$\chi_{LT,mod} = min(1.0, \frac{1}{\bar{\lambda}_{LT}^2}, \frac{\chi_{LT}}{f})$$

$$\chi_{LT,mod} = min(1.0, \frac{1}{1.52^2}, \frac{0.382}{1.0})$$

$$\chi_{LT,mod} = min(1.0, 0.433, 0.382)$$

$$\chi_{LT,mod} = 0.382$$

Final reduction factor for LTB

BS EN1993-1-1 Section 6.3.2.3(2) Equation 6.58

Final Moment Resistance with LTB Reduction Applied

Moment Resistance of beam accounting for lateral torsional buckling
$M_{b,Rd} = \chi_{LT,mod} * M_{c,Rd}$
$M_{b,Rd}$ = 0.382 * 792kNm
$M_{b,Rd}$ = 302kNm

Utilisation of the beam in lateral torsional buckling
= applied ULS moment / moment capacity accounting for LTB
= $M_{Ed} / M_{b,Rd}$
= 268kNm / 302kNm
= 89% --> OK

Deflections @ SLS

Deflections of the beam are calculated as part of the beam analysis

The utilisation of the beam in deflection is governed by the deflection limits set by the user.
Different deflection limits are used for:
Permanent (Dead) load deflection
Imposed (Live) load deflection
Total (Dead+Live) load deflection (also referred to as Total SLS load)

In this example the deflection limits have been set as follows:

Permanent (Dead) load deflection limit $\delta_{Gk,Limit}$ = SPAN / 200 = 6500mm / 200 = 32.5mm

Imposed (Live) load deflection limit $\delta_{Qk,Limit}$ = SPAN / 360 = 6500mm / 360 = 18.05mm

Total (Dead+Live) load deflection $\delta_{TOTAL,Limit}$ = SPAN / 200 = 6500mm / 200 = 32.5mm

Actual Permanent (Dead) load deflection δ_{Gk} = 5.208mm
Utilisation = δ_{Gk} / $\delta_{Gk,Limit}$
Utilisation = 5.208mm / 32.5mm
Utilisation = 16% --> OK

Actual Imposed (Live) load deflection δ_{Qk} = 3.28mm
Utilisation = δ_{Qk} / $\delta_{Qk,Limit}$
Utilisation = 3.28mm / 18.05mm
Utilisation = 18% --> OK

Actual Total load deflection δ_{TOTAL} = 8.48mm
Utilisation = δ_{TOTAL} / $\delta_{TOTAL,Limit}$
Utilisation = 8.48mm / 32.5mm
= 26% --> OK

Some guidance on vertical deflections at SLS is provided in the UK National Annex to BS EN1993-1-1 as shown below. However deflection limits will always vary based on the specific project and the client requirements. There may also be an absolute limit on the deflection i.e. 15mm for a brittle ceiling under total SLS load as an example.

NA.2.23 Vertical deflections [BS EN 1993-1-1:2005, 7.2.1(1)B]

The following table gives suggested limits for calculated vertical deflections of certain members under the characteristic load combination due to variable loads and should not include permanent loads. Circumstances may arise where greater or lesser values would be more appropriate. Other members may also need deflection limits.

On low pitch and flat roofs the possibility of ponding should be investigated.

Vertical deflection	
Cantilevers	Length/180
Beams carrying plaster or other brittle finish	Span/360
Other beams (except purlins and sheeting rails)	Span/200
Purlins and sheeting rails	To suit the characteristics of particular cladding

Vibration @ SLS

The equation provided on the spreadsheet to determine the natural frequency of the beam is based on the formula for a simply supported beam carrying a uniformly distributed load (UDL).

When the applied loading on the beam deviates from this simple UDL case the natural frequency will also deviate. Therefore this formula should only be used as an approximation. The formula also only provides the natural frequency of the beam in its 1st mode of vibration.

If you need to determine the true natural frequency of the beam for more complex loading arrangements and/or if you also require the subsequent modes of the vibration after the 1st mode I'd recommend running a finite element eigenvalue modal analysis. Most structural analysis packages have this ability built in.

Approximate natural frequency $$f_n = \frac{18}{\sqrt{\delta_{Gk} + 0.1\delta_{Qk}}}$$

Approximate natural frequency $$f_n = \frac{18}{\sqrt{5.208mm + 0.1 * 3.28mm}}$$

Approximate natural frequency $$f_n = 7.65H_z$$

You typically want to ensure the natural frequency of the beams 1st mode of vibration is at least 4Hz. This tends to be suitable for most domestic applications where the beam is supporting a residential space.

Where the beam is supporting a floor where more activity is anticipated (i.e. dance halls, gyms, stadia) more complex analysis is required. I'd recommend looking up the following documents if you find yourself in that situation.

Steel Construction Institute Publication P354 - Design of Floors for Vibration: A New Approach

Concrete Centre Publication CCIP-016 - A Design Guide for Footfall Induced Vibration of Structures ISBN: 1-904482-29-5 Published 2007

Natural frequency limit for current example $f_{n,limit}$ = 4H_z
Utilisation = $f_{n,limit}$ / f_n
Utilisation = 4Hz / 7.65Hz
Utilisation = 52% --> OK

Embodied Carbon

To calculate the embodied carbon of the beam you work out the total mass of the beam and then multiply by the embodied carbon factor for the type of steel you are using.

The embodied carbon factor can be found from the manufacturer or calculated using the following formula:

Construction Installation Carbon Factor (Including Site Wastage) = A5 + A5w

where:
A5 = (A13 + A4 + C2 + C34)

A5w = WF * (A13 + A4 + C2 + C34)

Waste Factor WF = (1 / (1 – WR)) – 1

A13 = Production Stage Carbon Factors (kgCO_2e / kg)
A4 = Transport Carbon Factors (from Manufacturing Plant to Site) (kgCO_2e / kg)
C2 = Transporting Wasted Material Away from Site (kgCO_2e / kg)
C34 = Processing and Disposal of the Waste Material (kgCO_2e / kg)
WR = Expected Percentage of Wastage on Site (%)

For the worked example the following values have been adopted:
A13 = 2.45kgCO_2e / kg
A4 = 0.032kgCO_2e / kg
C2 = 0.005kgCO_2e / kg
C34 = 0.013kgCO_2e / kg
WR = 1%

Waste Factor WF = (1 / (1 - 0.01)) - 1
Waste Factor WF = 0.01

A5w = 0.01 * (2.45 + 0.032 + 0.005 + 0.013)
A5w = 0.025kgCO_2e / kg

A5 = (2.45 + 0.032 + 0.005 + 0.013)
A5 = 2.5kgCO_2e / kg

A5 + A5w = 2.5 + 0.025
A5 + A5w = 2.525kgCO_2e / kg

Values taken from BS, 2020. Environmental product declaration (EPD) report of Steel Rails and Sections (including semi-fi nished long products). Gwent, BRC Limited. Available online at https://carbon.tips/rails (last accessed 30/04/20)

Also refer to IStructE How to calculate embodied carbon design guide for more information

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Beam Mass M = 98.3kg/m
Beam Length L = 6.5m
Beam total mass = M * L = 98.3kg/m * 6.5m = 638.95kg

Embodied Carbon = 638.95kg * 2.525kgCO2e/kg = 1613kgCO_2

NOTE: kgCO_2e / kg = kilograms of CO_2 equivalent per kg of material

Project	N/A	Steel Beam Design BS EN1993-1-1			
Client	N/A	Made by	Date	Job No	
Description	Worked Example 1	AL	N/A	N/A	
		Checked	Revision		
Steel Beam Analysis and Design v1.0		N/A	N/A		

1.0 - BEAM GEOMETRY (SPAN AND EFFECTIVE LENGTH)

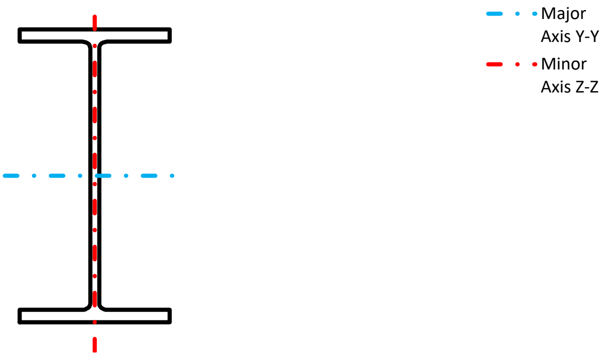
Overall Span of Beam L (m)	6.5	
Is Beam Laterally Restrained?	Laterally Unrestrained	
Restaint Conditions at Support A (Beginning of the Beam)	Bottom flange supported only with no positive connection	*1.2L + 2h*
Restaint Conditions at Support B (End of the Beam)	Bottom flange supported only with no positive connection	*1.2L + 2h*
Final Effective Length Used for Beam Design L_{eff} (m)	8.7344	

Effective Length Factor k = 1.34

2.0 - BEAM SECTION

Type of Section	UB
Beam Section	457x191x98

Is Beam Section Readily Available? --> YES



3.0 - SERVICEABILITY LIMIT STATE (SLS) - DEFLECTIONS & VIBRATION

Deflection Limit for Total Permanent Load (SPAN/___)	200	*32.5mm*
Deflection Limit for Total Imposed Load (SPAN/___)	360	*18.06mm*
Deflection Limit for Total Load (SPAN/___)	200	*32.5mm*
Include Approximate Natural Frequency Check?	YES	
Limiting Natural Frequency $f_{n,limit}$ (H_z)	4.000	

4.0 - MATERIAL PROPERTIES

Grade of Steel [BS EN1993-1-1 Table 3.1]	EN 10025-2: S355
Modulus of Elasticity E [BS EN1993-1-1 Section 3.2.6] (N/mm^2)	210000
Yield Strength of Steel Used for Capacity Checks f_y (N/mm^2)	355
Poissons Ratio v [BS EN1993-1-1 Section 3.2.6]	0.3
Shear Modulus G (N/mm^2) [BS EN1993-1-1 Section 3.2.6]	80769
ε = √235 / f_y [BS EN1993-1-1 Table 5.2]	0.814

5.0 - ULTIMATE LIMIT STATE (ULS) LOAD FACTORS

ULS Factor for Permanent (Dead) Load $G_{k,factor}$	1.35
ULS Factor for Imposed (Live) Load $Q_{k,factor}$	1.5

6.0 - BEAM SECTIONAL PROPERTIES (FROM SCI BLUE BOOK ONLINE)

Area of Section A (mm^2)	12500
Mass Per Metre Length M (kg/m)	98.3
Depth of Section h (mm)	467.2
Width of Section b (mm)	192.8
Thickness of Web t_w (mm)	11.4
Thickness of Flange t_f (mm)	19.6
Root Radius r (mm)	10.2
Depth Between Fillets d (mm)	407.6
Second Moment of Area - Major Axis I_y (mm^4)	457000000
Second Moment of Area - Minor Axis I_z (mm^4)	23500000
Radius of Gyration - Major Axis i_y (mm)	191
Radius of Gyration - Minor Axis i_z (mm)	43.3
Elastic Modulus - Major Axis $W_{el,y}$ (mm^3)	1960000
Elastic Modulus - Minor Axis $W_{el,z}$ (mm^3)	243000
Plastic Modulus - Major Axis $W_{pl,y}$ (mm^3)	2230000
Plastic Modulus - Minor Axis $W_{pl,z}$ (mm^3)	379000
Buckling Parametre u	0.881
Torsional Index x	25.8
Warping Constant I_w (mm^6)	1180000000000
Torsional Constant Inertia I_t (mm^4)	1210000

7.0 - APPLIED BEAM LOADING (UNFACTORED SLS LOADING)

Load Type	Load Description	Load 1 (SLS)		Distance to Beginning of Load (dim a) (m)	Length of UDL Line Load (dim b) (m)	Load 2 (SLS)	
		Permanent	Imposed			Permanent	Imposed
Self Weight	Include Beam Self Weight	0.964	---	---	---	---	---
Full Length Uniformly Distributed Line Load (UDL)	UDL1	10.00	6.50	---	---	---	---
				---	---	---	---
				---	---	---	---
				---	---	---	---
				---	---	---	---
Point Load	P1	7.00	2.50	3.5	---	---	---
	P2	4.00	8.00	4.5	---	---	---
					---	---	---
					---	---	---
					---	---	---
					---	---	---
					---	---	---
					---	---	---
					---	---	---
					---	---	---
Partial Trapezoidal Line Load	UDL2	5.00	3.00	1.5	4	15.00	9.00

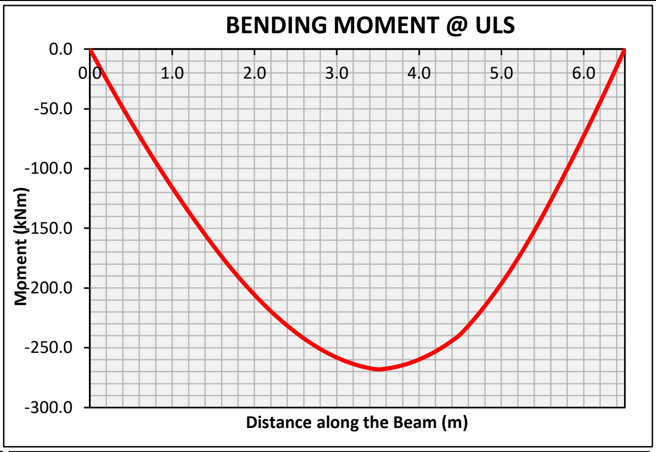
8.0 - INTERNAL FORCES & REACTION FORCES

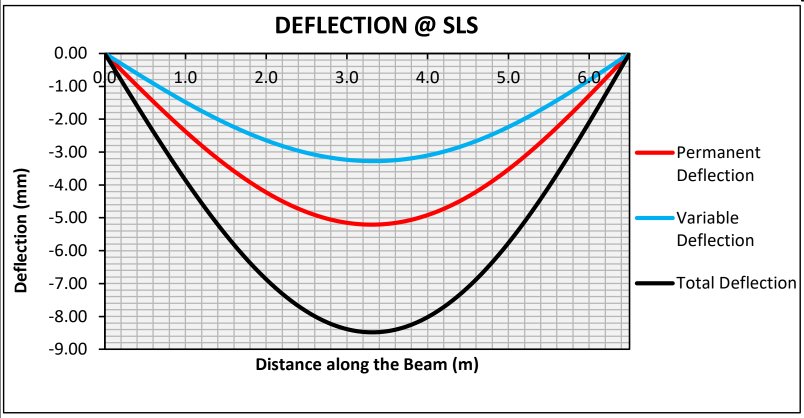
8.1 - INTERNAL FORCES

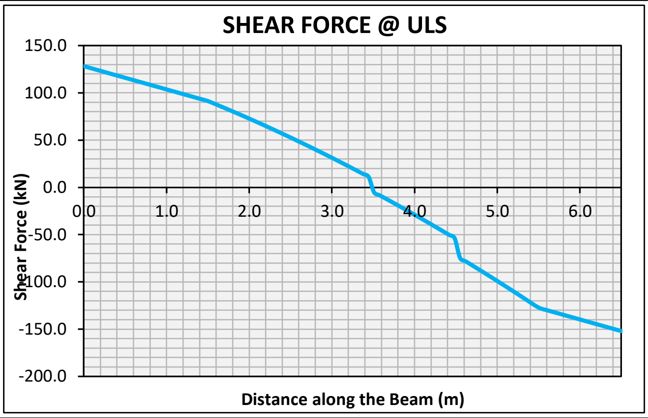
Override Auto Calculated Forces and use Manual Values?	NO
Maximum Bending Moment M_{Ed} (kNm)	268.140
Distance along the beam to Max Bending Moment x_{MEd} (m)	3.510
Maximum Shear Force V_{Ed} (kN)	152.024
Distance along the beam to Max Shear Force x_{VEd} (m)	7.000

8.2 - REACTION FORCES

	Start of Beam		End of Beam	
Permanent Load (SLS)	56.506		65.762	
Imposed Load (SLS)	34.587		42.163	
Total Load (SLS)	91.092	0.000	107.926	0.000
Total Load (ULS)	128.163	0.000	152.024	0.000







9.0 - SECTION CLASS (CHECKING FOR LOCAL BUCKLING OF WEB AND FLANGES)

Section Class of Web (Web in Bending)	1	$d/t_w \leq 72\varepsilon$: Class 1 $72\varepsilon < d/t_w \leq 83\varepsilon$: Class 2 $83\varepsilon < d/t_w \leq 124\varepsilon$: Class 3 where d = depth between fillets Table 5.2 (Sheet 1 of 3)
Section Class of Flanges (Flange in Uniform Compression)	1	$c/t_f \leq 9\varepsilon$: Class 1 $9\varepsilon < c/t_f \leq 10\varepsilon$: Class 2 $10\varepsilon < c/t_f \leq 14\varepsilon$: Class 3 where $c = (b/2) - (t_w/2) - r$ Table 5.2 (Sheet 2 of 3)
Final Section Class	1	Final Section Class is the Maximum of the Web and Flange Classes

10.0 - PARTIAL MATERIAL SAFETY FACTORS

Factor for Resistance of Cross Section no matter the section class γ_{M0}	1.0	Section 6.1 NOTE 2B & UK National Annex Section NA.2.15
Factor for Resistance of Members to Instability γ_{M1}	1.0	Section 6.1 NOTE 2B & UK National Annex Section NA.2.15
Factor for Resistance of Joints γ_{M2}	1.1	Section 6.1 NOTE 2B & UK National Annex Section NA.2.15

11.1 - CHECK MAJOR AXIS SHEAR - WEB BUCKLING @ ULTIMATE LIMIT STATE (ULS)

Height of the Beam Web between the flanges h_w (mm)	428	$h_w = h - 2t_f$
η Factor for Shear Area	1.0	BS EN1993-1-1 & 1-5 Recommended Value
Does Shear Buckling of Web Have to Be Considered?	NO	$h_w / t_w > 72\varepsilon / \eta$

11.2 - CHECK MAJOR AXIS SHEAR @ ULTIMATE LIMIT STATE (ULS)

Maximum Applied Shear Force V_{Ed} (kN)	152.024	From Shear Force Diagram
Shear Area A_v (mm^2)	5566	$A_v = \max\{ \eta * (h - 2t_f) * t_w ;\ A - (2 * b * t_f) + (t_w + 2r) * t_f \}$ Section 6.2.6(3)
Plastic Shear Resistance of Section $V_{pl,Rd}$ (kN)	1141	$V_{pl,Rd} = (A_v * f_y) * (1 / \sqrt{3}) * (1 / \gamma_{M0})$ Section 6.2.6(2) Eq - 6.18
Utilisation %	13%	$V_{Ed} / V_{pl,Rd}$

12 - CHECK MAJOR AXIS BENDING @ ULTIMATE LIMIT STATE (ULS)

Maximum Applied Bending Moment M_{Ed} (kNm)	268.140	From Bending Moment Diagram
Basic Moment Resistance of Section $M_{c,Rd,basic}$ (kNm)	791.650	$M_{c,Rd,basic} = M_{pl,Rd} = W_{pl,y} * f_y * (1 / \gamma_{M0})$ --> for class 1 or 2 sections $M_{c,Rd,basic} = M_{el,Rd} = W_{el,y} * f_y * (1 / \gamma_{M0})$ --> for class 3 sections Section 6.2.5(2) Eq - 6.13 & 6.13
Reduce Bending Resistance Based on Max Shear Force?	NO	Is $V_{Ed} \geq 0.5V_{pl,Rd}$? Section 6.2.8(2)
Reduction Factor for Shear Acting with Bending p	0.000	if required $p = [(2V_{Ed} / V_{pl,Rd}) - 1]^2$ Section 6.2.8(2) Eq - 6.29
Final Moment Resistance of Section including Reductions $M_{c,Rd}$ (kNm)	791.650	$M_{c,Rd} = [(W_{pl,y} - W_{pl,y,web}) + (W_{pl,y,web}*(1-p))] * f_y * (1/\gamma_{M0})$ --> for class 1 or 2 $M_{c,Rd} = [(W_{el,y} - W_{el,y,web}) + (W_{el,y,web}*(1-p))] * f_y * (1/\gamma_{M0})$ --> for class 3 $W_{pl,y,web} = t_w h^2/4$ $W_{el,y,web} = t_w h^2/6$ Section 6.2.8(2) Eq - 6.30
Utilisation %	34%	$M_{Ed} / M_{c,Rd}$

13 - CHECK LATERAL TORSIONAL BUCKLING @ ULTIMATE LIMIT STATE (ULS) (GENERAL CASE FROM BS EN1993-1-1 SECTION 6.3.2.2)

13.1 - DESTABILISING LOADS

Is the Applied Transverse Load Destabilising?	NO	
Destabilising Load Factor D		D = 1.0 No Destabilising Load

13.2 - C1 FACTOR

Calculation of C_1 Factors	AUTO	$C_1 = \min\{ 2.5 , 4M_{Ed} / \sqrt{[M_{Ed}^2 + 4M_a^2 + 7M_b^2 + 4M_c^2]}\}$ - Wong & Driver			
C_1 Factor (manual user input)		M_{Ed}	M_a	M_b	M_c
Final C_1 Factor	1.157	268	176kNm @ 1.625m	265kNm @ 3.25m	208kNm @ 4.875m

13.3 - ALLOWANCE FOR IN-PLANE CURVATURE OF THE BEAM PRIOR TO BUCKLING (REFER TO SCI P360 PAGE 11)

Allowance for In-Plane Curvature of the Beam Prior to Buckling	INCLUDE	This is a beneficial factor, omitting this factor is conservative	
g Factor (1.0 is conservative)	0.974	$g = \sqrt{[1 - (I_z / I_y)]}$	SCI P360 Section 2.2.1, Page 11

13.4 - REDUCED MOMENT RESISTANCE FOR LATERAL TORSIONAL BUCKLING (LTB) (ROLLED OR WELDED SECTIONS BS EN1993-1-1 SECTION 6.3.2.3)

Elastic Critical Buckling Moment for Lateral Torsional Buckling M_{cr} (kNm)	342.013	$M_{cr} = C_1 \frac{\pi^2 EI_z}{g(kL)^2} \sqrt{\frac{I_w}{I_z} + \frac{(kL)^2 GI_t}{\pi^2 EI_z}}$	Steel Construction Institute Publication P360 - Eq 2.14 (Simplified)
Slenderness for LTB $\bar{\lambda}_{LT}$	1.521	$\bar{\lambda}_{LT} = D\sqrt{[(W_{pl,y} * f_y) / M_{cr}]}$ --> For Class 1&2 Section $\bar{\lambda}_{LT} = D\sqrt{[(W_{el,y} * f_y) / M_{cr}]}$ --> For Class 3 Section	Section 6.3.2.2(1)
Buckling Curve	c	Table 6.5 Using Rolled I-Sections	
Imperfection Factor α_{LT}	0.490	Table 6.3	
Additional Slenderness Factor $\bar{\lambda}_{LT,0}$	0.400	from UK National Annex (Typically 0.4)	Section 6.3.2.3(1) & NA.2.17
Modification Factor β	0.750	from UK National Annex (Typically 0.75)	Section 6.3.2.3(1) & NA.2.17
ϕ_{LT}	1.643	$\phi_{LT} = 0.5[1 + \alpha_{LT}(\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}) + \beta\bar{\lambda}_{LT}^2]$	Section 6.3.2.3(1)
Initial Reduction Factor for LTB χ_{LT}	0.381	$\chi_{LT} = \min\{ 1.0 \mid 1 / [\phi_{LT} + \sqrt{(\phi_{LT}^2 - \beta\bar{\lambda}_{LT}^2)}] \mid 1 / \bar{\lambda}_{LT}^2\}$	Eq - 6.57
Slenderness Correction Factor k_c	0.930	$k_c = 1 / \sqrt{C_1}$	Table 6.6 & NA Section NA.2.18
Factor to Account for Moment Distribution Between the Lateral Restraints f	1.000	$f = \min\{ 1.0 \mid 1 - 0.5(1 - k_c)[1 - 2(\bar{\lambda}_{LT} - 0.8)^2] \}$	Section 6.3.2.3(2)
Final Reduction Factor for LTB $\chi_{LT,mod}$	0.381	$\chi_{LT,mod} = \min\{ 1.0 \mid 1/\bar{\lambda}_{LT}^2 \mid \chi_{LT} / f \}$	Section 6.3.2.3(2) Eq - 6.58
Moment Resitance of Section Accounting for LTB $M_{b,Rd}$ (kNm)	301.710		Section 6.3.2.1(3) Eq - 6.55
Utilisation %	**89%**	$M_{Ed} / M_{b,Rd}$	

14.0 - DEFLECTIONS @ SERVICABILITY LIMIT STATE (SLS)

14.1 - PERMANENT LOAD DEFLECTION

Maximum Deflection from Permanent Load δ_{Gk} (mm)	5.208	From Deflection Analysis
Deflection Limit for Permanent Load $\delta_{Gk,Limit}$ (mm)	32.500	User Input
Utilisation %	**16%**	$\delta_{Gk} / \delta_{Gk,Limit}$

14.2 - IMPOSED LOAD DEFLECTION

Maximum Deflection from Imposed Load δ_{Qk} (mm)	3.28	From Deflection Analysis
Deflection Limit for Imposed Load $\delta_{Qk,Limit}$ (mm)	18.056	User Input
Utilisation %	**18%**	$\delta_{Qk} / \delta_{Qk,Limit}$

14.3 - TOTAL DEFLECTION

Maximum Deflection from Total Load δ_{TOTAL} (mm)	8.48	From Deflection Analysis
Deflection Limit for Total Load $\delta_{TOTAL,Limit}$ (mm)	32.500	User Input
Utilisation %	**26%**	$\delta_{TOTAL} / \delta_{TOTAL,Limit}$

15.0 - NATURAL FREQUENCY - APPROXIMATE ASSESSMENT USING FORMULA FOR UDL ON SIMPLY SUPPORTED BEAM

Approximate Natural Frequency [1^{st} Mode of Vibration] f_n (H_z)	7.650	$f_n = 18 / \sqrt{(\delta_{Gk} + 0.1\delta_{Qk})}$
Approximate Natural Frequency $f_{n,limit}$ (H_z)	4.000	User Input
Utilisation %	**52%**	$f_{n,limit} / f_n$

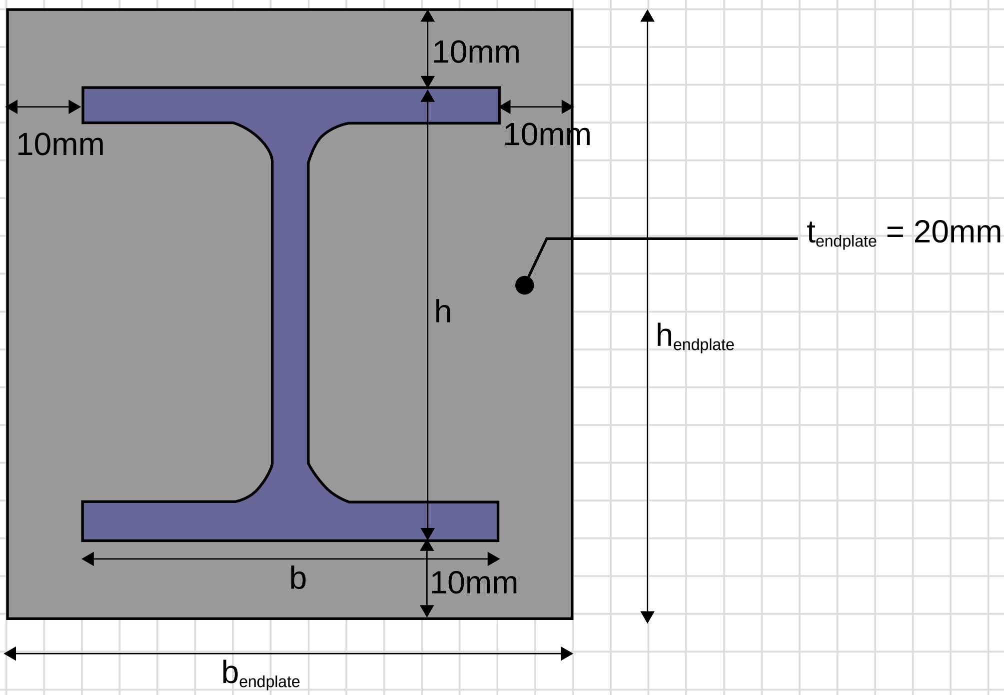
DESIGN STATUS	**PASS**
OVERALL UTILISATION	**89%**

16.0 - EMBODIED CARBON		
16.1 - EMBODIED CARBON FACTORS FOR STEEL		
Type of Steel Adopted:	UK Open Sections: British Steel EPD	
Production Stage Carbon Factor - A13 ($kgCO_2e/kg$)	2.450	User input (Embodied Carbon Database)
Transport Carbon Factors (from Manufacturing Plant to Site) - A4 ($kgCO_2e/kg$)	0.032	User input (Embodied Carbon Database)
Transporting Wasted Material Away from Site - C2 ($kgCO_2e/kg$)	0.005	User input (Embodied Carbon Database)
Processing and Disposal of the Waste Material - C34 ($kgCO_2e/kg$)	0.013	User input (Embodied Carbon Database)
Expected Percentage of Wastage on Site - WR (%)	1.00%	User input (Embodied Carbon Database)
Waste Factor Based on Expected % Waste Rate During Construction - WF	0.010	WF = (1 / (1 – WR)) – 1
Construction Installation Carbon Factor (Inc Site Wastage)	**2.525**	A5 + A5w = (WF + 1) * (A13 + A4 + C2 + C34)
16.2 - EMBODIED CARBON FOR SELECTED STEEL BEAM		
Total Mass of Steel Beam M_{total} (kg)	638.950	M_{total} = M * L (UB457x191x98 : 6.5m long)
Embodied Carbon for Beam EC_{beam} ($kgCO_2e/kg$)	**1614**	EC_{beam} = M_{total} * (A5 + A5w)
16.3 - ALLOWANCE FOR WELDED END PLATES		
Approximate Width of Welded Endplate $B_{endplate}$ (mm)	225	$B_{endplate}$ = (b + 10mm + 10mm) --> rounded up to nearest 25mm
Approximate Height of Welded Endplate $H_{endplate}$ (mm)	500	$H_{endplate}$ = (h + 10mm + 10mm) --> rounded up to nearest 25mm
Approximate Thickness Welded Endplate $t_{endplate}$ (mm)	20	$t_{endplate}$ = 20mm as standard
Mass of a Single Welded Endplate $M_{endplate}$ (kg)	17.663	$M_{endplate}$ = $B_{endplate}$ * $H_{endplate}$ * $t_{endplate}$ * 7850kg/m^3
Mass of Both Welded Endplates $\Sigma M_{endplate}$ (kg)	35.325	$\Sigma M_{endplate}$ = 2 * $M_{endplate}$
Embodied Carbon for Welded Endplates $EC_{endplate}$ ($kgCO_2e/kg$)	**89.2**	$EC_{endplate}$ = $\Sigma M_{endplate}$ * (A5 + A5w)
16.4 - TOTAL EMBODIED CARBON (INC ENDPLATES)		
Embodied Carbon for Entire Beam EC_{total} ($kgCO_2e/kg$)	**1703**	EC_{total} = EC_{beam} + $EC_{endplate}$

Embodied Carbon (Continued)

The spreadsheet also accounts for welded endplates to each end of the beam in the embodied carbon assessment.

The welded endplates are assumed to be 20mm thick and are also assumed to be 10mm wider than the beam on all edges (and rounded to up to the nearest 25mm).



$h_{endplate}$ = roundup (h + 10mm + 10mm)
$h_{endplate}$ = roundup (467.2mm + 10mm + 10mm)
$h_{endplate}$ = 500mm

$b_{endplate}$ = roundup (b + 10mm + 10mm)
$b_{endplate}$ = roundup (192.8mm + 10mm + 10mm)
$b_{endplate}$ = 225mm

Mass of a single endplate
$M_{endplate} = (h_{endplate} * b_{endplate} * t_{endplate}) * 7850kg/m^3$
$M_{endplate}$ = (500mm * 225mm * 20mm) * $7850kg/m^3$
$M_{endplate}$ = 17.7kg (per endplate)

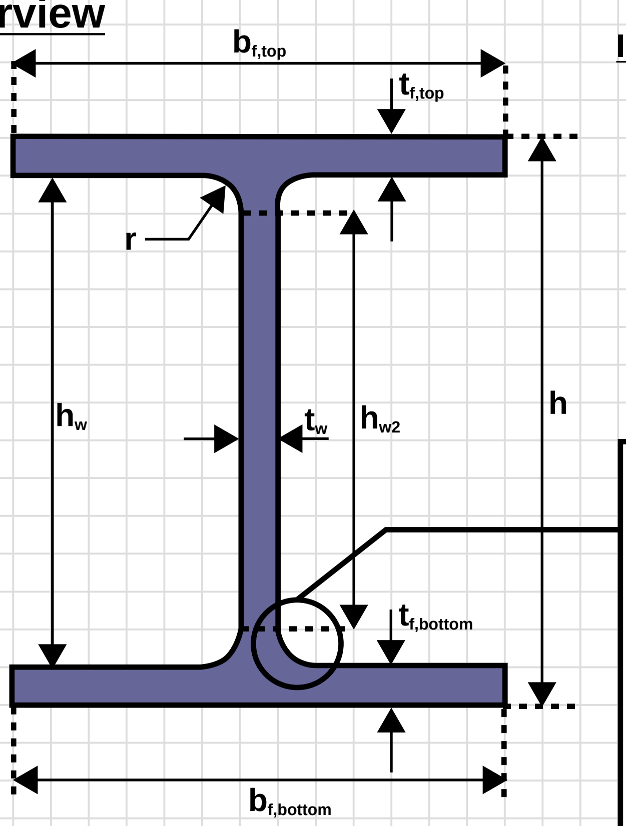
Mass of both endplates
$\Sigma M_{endplate} = 2 * M_{endplate}$
$\Sigma M_{endplate}$ = 2 * 17.7kg
$\Sigma M_{endplate}$ = 35.4kg

Embodied carbon of both endplates
= ΣMendplate * 2.525kgCO_2e / kg
= 35.4kg * 2.525kgCO_2e / kg
= 89.4kgCO_2

Total embodied carbon of the beam and the endplates
= 1613kgCO_2 + 89.4kgCO_2
= 1702kgCO_2

	Design Spreadsheet: **Steel Beam Analysis and Design**	Date: **15/05/22**
	Subtitle: **Custom Section Properties**	By: **AL**
		Version: **1**

Overview

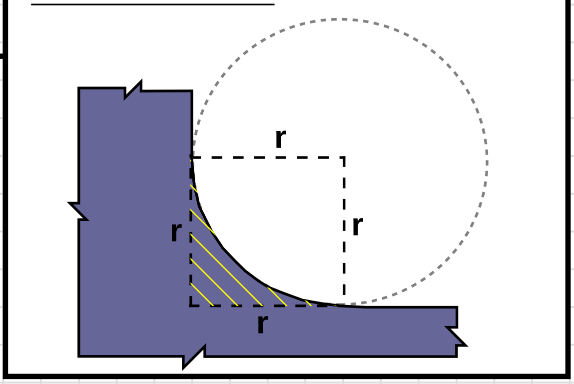


Input Dimensions

h = height of beam (mm)
$b_{f,top}$ = width of top flange (mm)
$b_{f,bottom}$ = width of bottom flange (mm)
$t_{f,top}$ = thickness of top flange (mm)
$t_{f,bottom}$ = thickness of bottom flange (mm)
t_w = thickness of web (mm)
r = root radius of corners (mm)

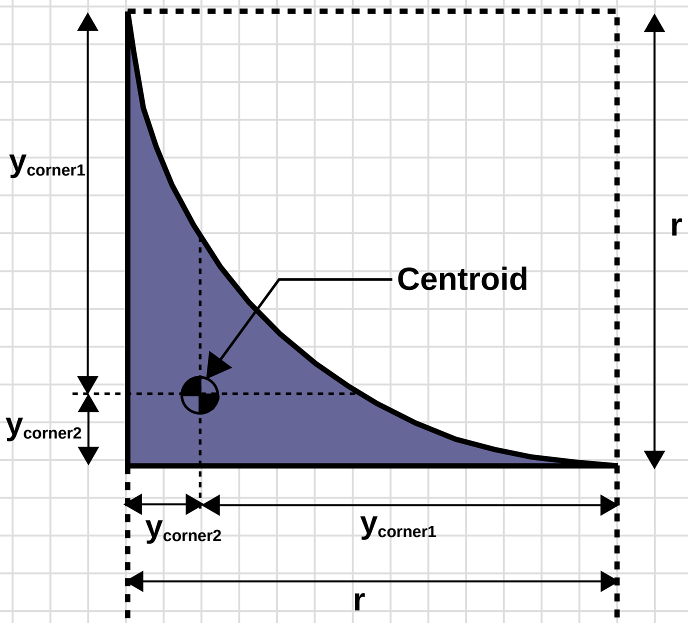
$h_w = h - t_{f,top} - t_{f,bottom}$ (mm)
$h_{w2} = h - t_{f,top} - t_{f,bottom} - 2r$ (mm)

Filleted Corners



Calculate Second Moments of Area

Centroid & Second Moment of Area of the Filleted Corner



Centroid

$y_{corner1} = r / (6 - 1.5\pi)$

$y_{corner2} = r - [r / (6 - 1.5\pi)]$

Second Moment of Area (About the Centroidal Axis)

$$I_{corner} = 4r^4\left(\frac{1}{12} - \frac{\pi}{64}\right) - y_{corner1}^2\left(r^2 - \frac{\pi r^2}{4}\right)$$

units are Length⁴, usually mm⁴

Area of Filleted Corner

$$A_{corner} = r^2 - \frac{\pi r^2}{4}$$

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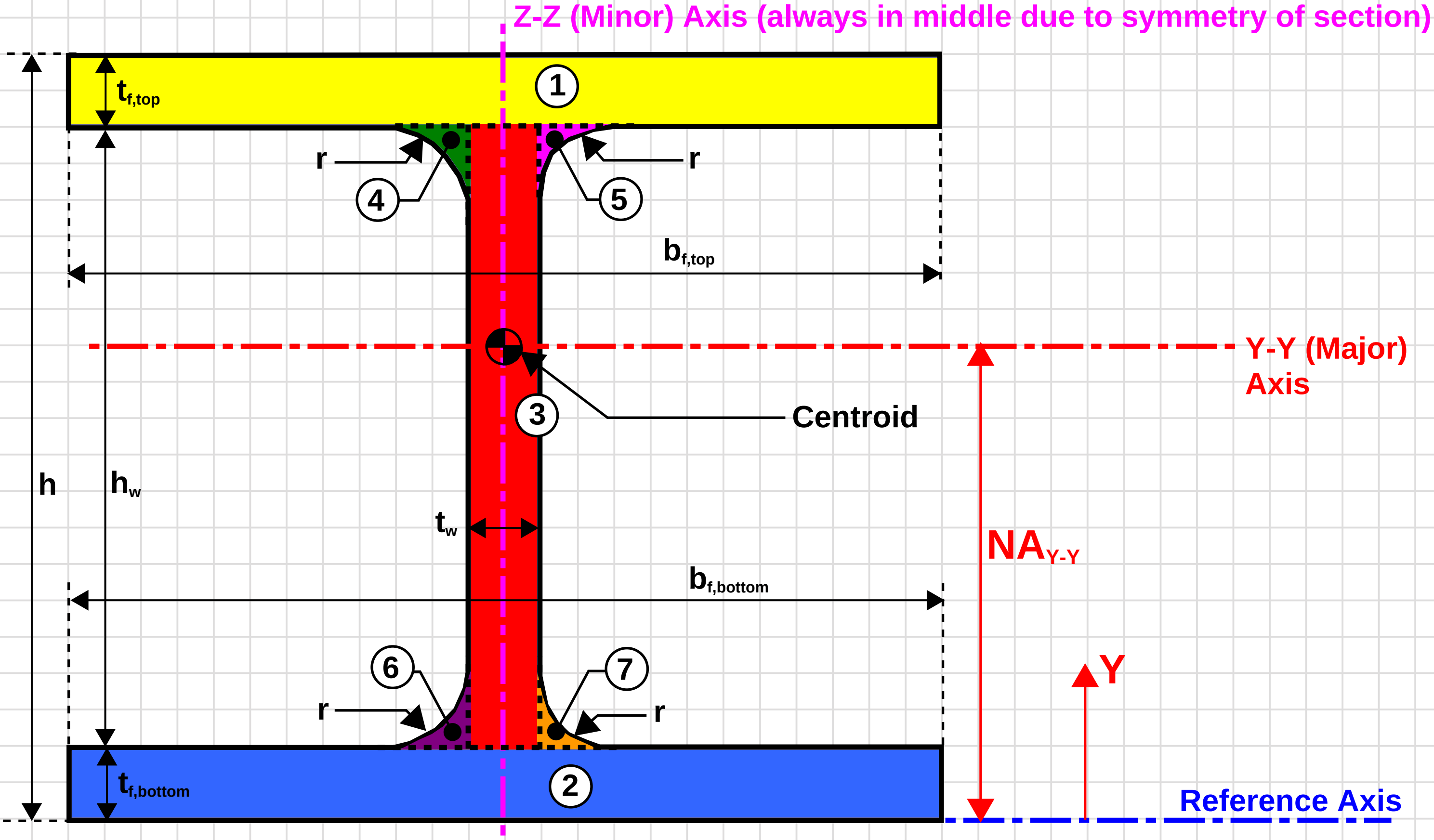
Subtitle: Custom Section Properties	By: AL
	Version: 1

Centroid of Overall Section (Major Y-Y Axis)



Subdivided Area	Area A (mm^2)	Distance to Centroid from Reference Axis Y (mm)	A * Y (mm^3)
1 - Top Flange	$b_{f,top} * t_{f,top}$	$h - (t_{f,top} / 2)$	$[b_{f,top} * t_{f,top}] * [h - (t_{f,top} / 2)]$
2 - Bottom Flange	$b_{f,bottom} * t_{f,bottom}$	$t_{f,bottom} / 2$	$[b_{f,bottom} * t_{f,bottom}] * [t_{f,bottom} / 2]$
3 - Web	$h_w * t_w$	$t_{f,bottom} + (h_w / 2)$	$[h_w * t_w] * [t_{f,bottom} + (h_w / 2)]$
4 - Filleted Corner 1	$r^2 - 0.25\pi r^2$	$h - t_{f,top} - y_{corner2}$	$[r^2 - 0.25\pi r^2] * [h - t_{f,top} - y_{corner2}]$
5 - Filleted Corner 2	$r^2 - 0.25\pi r^2$	$h - t_{f,top} - y_{corner2}$	$[r^2 - 0.25\pi r^2] * [h - t_{f,top} - y_{corner2}]$
6 - Filleted Corner 3	$r^2 - 0.25\pi r^2$	$t_{f,bottom} + y_{corner2}$	$[r^2 - 0.25\pi r^2] * [t_{f,bottom} + y_{corner2}]$
7 - Filleted Corner 4	$r^2 - 0.25\pi r^2$	$t_{f,bottom} + y_{corner2}$	$[r^2 - 0.25\pi r^2] * [t_{f,bottom} + y_{corner2}]$
TOTALS	$\Sigma(A)$	**TOTALS**	$\Sigma(A * y)$

$$NA_{Y-Y} = \Sigma(A * y) / \Sigma(A)$$ ---> *units are Length, usually mm*

Centroid of Overall Section (Minor Z-Z Axis)

Because the section is symmetric about the Z-Z axis the centroid will always be in the middle of the web as shown in the diagram above

Design Spreadsheet: Steel Beam Analysis and Design	Date: 15/05/22
	By: AL
Subtitle: Second Moments of Area Derivation	Version: 1

Second Moment of Area of Overall Section About Y-Y (Major) Axis (Parallel Axis Theorem)

Subdivided Area	Second Moment of Area about Centroid of Subdivided Element I_y (mm^4)	Distance from Neutral Axis to Centroid of Subdivided Element Y_{bar} (mm)	(Y_{bar} for each element)^2 *(Area of each element) $Y_{bar}^2 * A$
1 - Top Flange	$b_{f,top} * (t_{f,top})^3 * (1/12)$	NA_{Y-Y} - [h - ($t_{f,top}$ / 2)]	$(Y_{bar})^2 * b_{f,top} * t_{f,top}$
2 - Bottom Flange	$b_{f,bottom} * (t_{f,bottom})^3 * (1/12)$	NA_{Y-Y} - [$t_{f,bottom}$ / 2]	$(Y_{bar})^2 * b_{f,bottom} * t_{f,bottom}$
3 - Web	$t_w * (h_w)^3 * (1/12)$	NA_{Y-Y} - [$t_{f,bottom}$ + (h_w / 2)]	$(Y_{bar})^2 * h_w * t_w$
4 - Filleted Corner 1	$I_{corner} = 4r^4(\frac{1}{12} - \frac{\pi}{64}) - y^2_{corner1}(r^2 - \frac{\pi r^2}{4})$	NA_{Y-Y} - [h - $t_{f,top}$ - $y_{corner2}$]	$(Y_{bar})^2 * (r^2 - 0.25\pi r^2)$
5 - Filleted Corner 2	$I_{corner} = 4r^4(\frac{1}{12} - \frac{\pi}{64}) - y^2_{corner1}(r^2 - \frac{\pi r^2}{4})$	NA_{Y-Y} - [h - $t_{f,top}$ - $y_{corner2}$]	$(Y_{bar})^2 * (r^2 - 0.25\pi r^2)$
6 - Filleted Corner 3	$I_{corner} = 4r^4(\frac{1}{12} - \frac{\pi}{64}) - y^2_{corner1}(r^2 - \frac{\pi r^2}{4})$	NA_{Y-Y} - [$t_{f,bottom}$ + $y_{corner2}$]	$(Y_{bar})^2 * (r^2 - 0.25\pi r^2)$
7 - Filleted Corner 4	$I_{corner} = 4r^4(\frac{1}{12} - \frac{\pi}{64}) - y^2_{corner1}(r^2 - \frac{\pi r^2}{4})$	NA_{Y-Y} - [$t_{f,bottom}$ + $y_{corner2}$]	$(Y_{bar})^2 * (r^2 - 0.25\pi r^2)$
TOTALS	$\Sigma(I_y)$	**TOTALS**	$\Sigma(Y_{bar}^2 * A)$

$\mathbf{I_{Y-Y} = \Sigma(I_y) + \Sigma(Y_{bar}^2 * A)}$ *---> units are Length^4, usually mm^4*

Second Moment of Area of Overall Section About Z-Z (Minor) Axis (Parallel Axis Theorem)

In this instance to simplify the calculation we'll take the reference axis through the middle of the section because we know that this is where the centroid lies. This means the parallel axis calculation simplifies because the centroids of the flanges and web coincide with the reference axis.

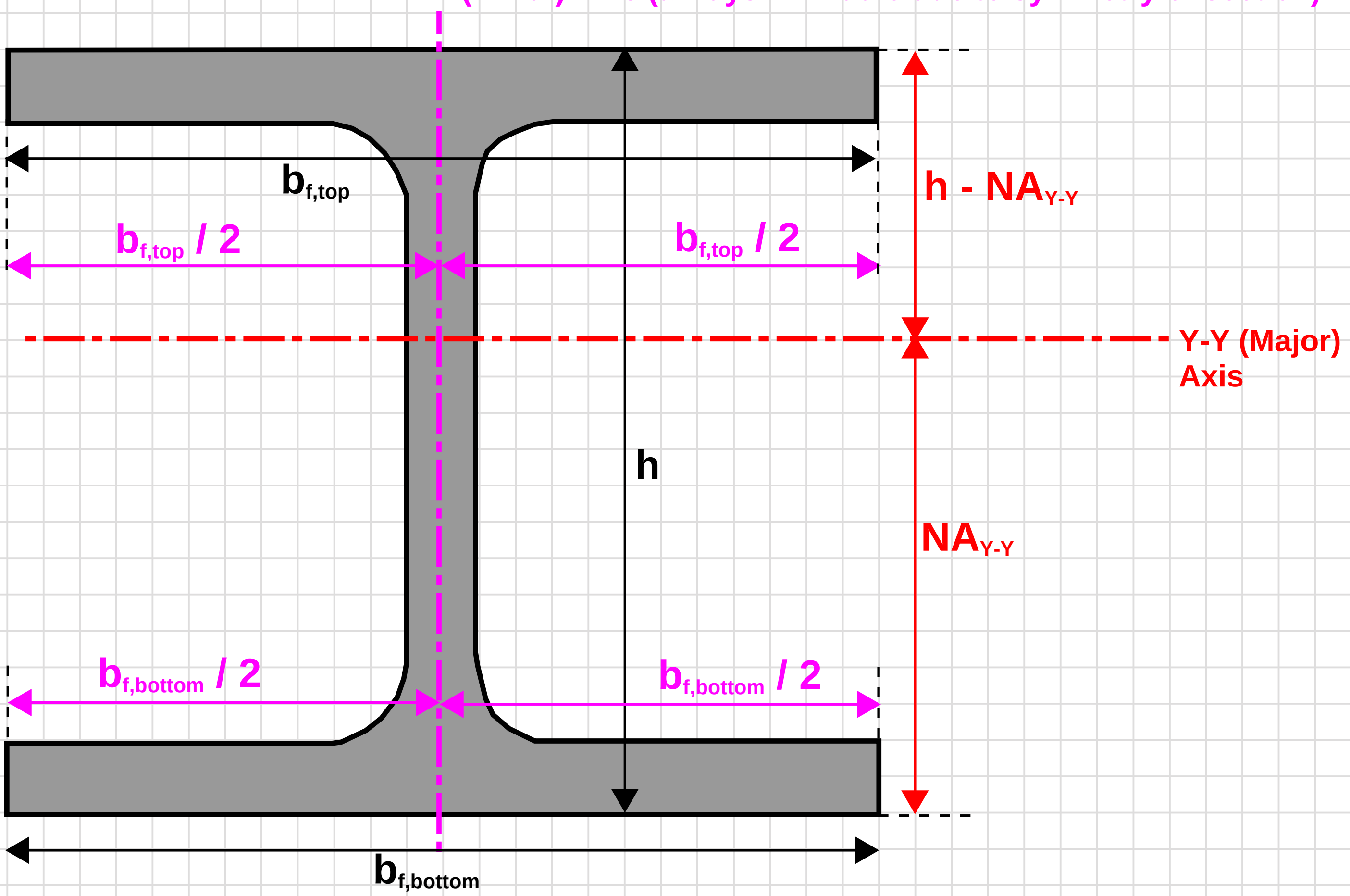
Subdivided Area	Second Moment of Area about Centroid of Subdivided Element I_z (mm^4)	Distance from Neutral Axis to Centroid of Subdivided Element Z_{bar} (mm)	(Z_{bar} for each element)^2 * (Area of each element) $Z_{bar}^2 * A$
1 - Top Flange	$(t_{f,top}) * (b_{f,top})^3 * (1/12)$	0	0
2 - Bottom Flange	$(t_{f,bottom}) * (b_{f,bottom})^3 * (1/12)$	0	0
3 - Web	$h_w * (t_w)^3 * (1/12)$	0	0
4 - Filleted Corner 1	$I_{corner} = 4r^4(\frac{1}{12} - \frac{\pi}{64}) - y^2_{corner1}(r^2 - \frac{\pi r^2}{4})$	$(-t_w / 2) - y_{corner2}$	$(Z_{bar})^2 * (r^2 - 0.25\pi r^2)$
5 - Filleted Corner 2	$I_{corner} = 4r^4(\frac{1}{12} - \frac{\pi}{64}) - y^2_{corner1}(r^2 - \frac{\pi r^2}{4})$	$(t_w / 2) + y_{corner2}$	$(Z_{bar})^2 * (r^2 - 0.25\pi r^2)$
6 - Filleted Corner 3	$I_{corner} = 4r^4(\frac{1}{12} - \frac{\pi}{64}) - y^2_{corner1}(r^2 - \frac{\pi r^2}{4})$	$(-t_w / 2) - y_{corner2}$	$(Z_{bar})^2 * (r^2 - 0.25\pi r^2)$
7 - Filleted Corner 4	$I_{corner} = 4r^4(\frac{1}{12} - \frac{\pi}{64}) - y^2_{corner1}(r^2 - \frac{\pi r^2}{4})$	$(t_w / 2) + y_{corner2}$	$(Z_{bar})^2 * (r^2 - 0.25\pi r^2)$
TOTALS	$\Sigma(I_Z)$	**TOTALS**	$\Sigma(Z_{bar}^2 * A)$

$\mathbf{I_{Z-Z} = \Sigma(I_Z) + \Sigma(Z_{bar}^2 * A)}$ *---> units are Length^4, usually mm^4*

Calculate Elastic Section Modulus

Elastic Section Modulus About Major Y-Y Axis

The elastic section modulus is found from the engineers bending equation and is defined as:
"The second moment of area divided by the minimum distance from the neutral axis to the extreme fibre"



$$W_{el,y} = \frac{I_{Y-Y}}{min(NA_{Y-Y}\ ;\ h - NA_{Y-Y})}$$ ---> *units are Length³, usually mm³*

Elastic Section Modulus About Minor Z-Z Axis

For the minor axis we can again leverage the fact that we know that the neutral axis will lie along the centreline of the section. Therefore the distance from this axis to the extreme fibre is just the minimum width of the section divided by 2:

$$W_{el,z} = \frac{I_{Z-Z}}{min(\frac{b_{f,top}}{2}\ ;\ \frac{b_{f,bottom}}{2})}$$ ---> *units are Length³, usually mm³*

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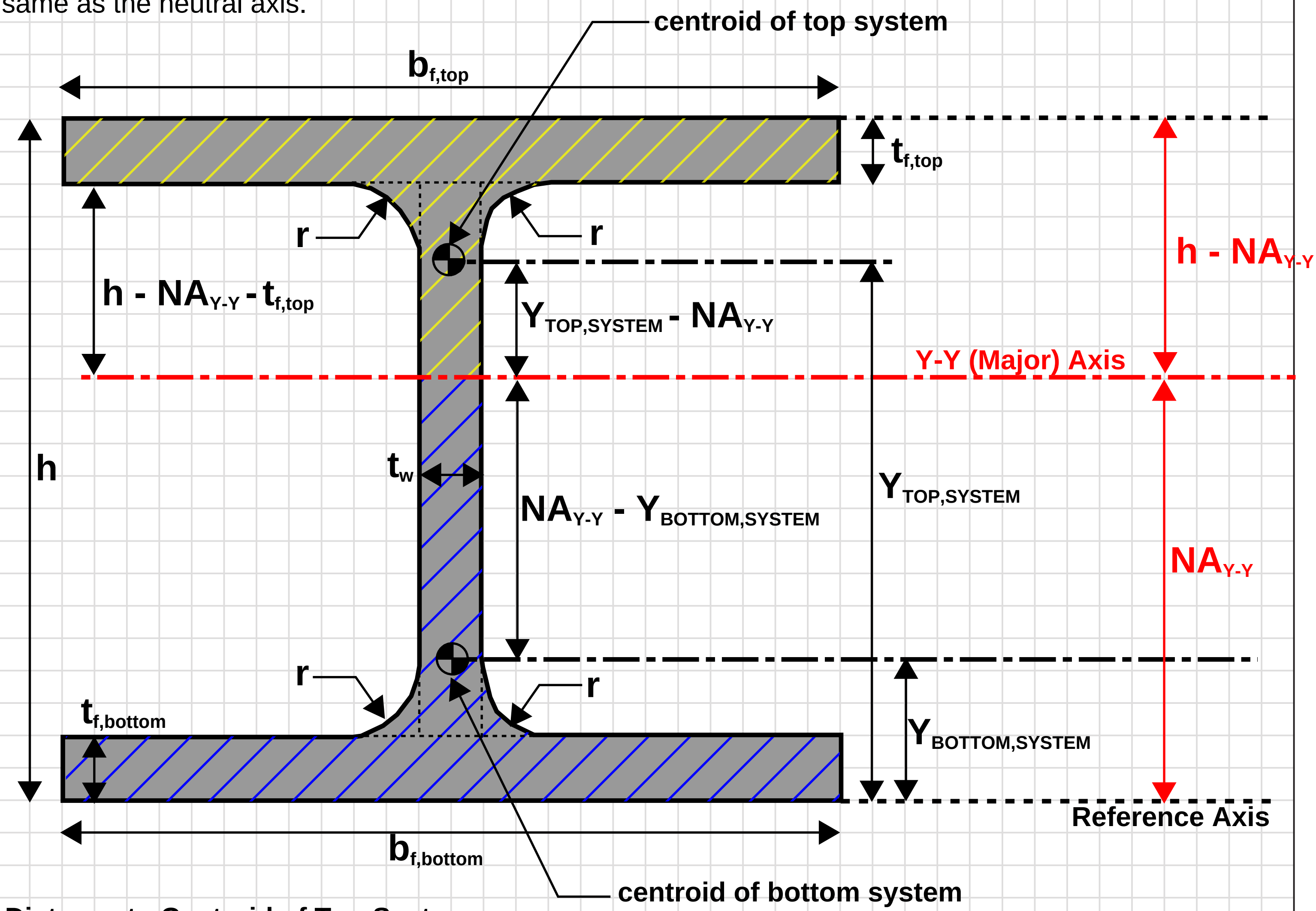
Calculate Plastic Section Modulus

Plastic Section Modulus About Major Y-Y Axis

The plastic section modulus is defined as follows:
"The plastic section modulus is the sum of the areas of the cross section on each side of the plastic neutral axis (which may or may not be equal) multiplied by the distance from the local centroids of the two areas to the plastic neutral axis"
In this instance because the section is all made from the same material the plastic neutral axis is the same as the neutral axis.



Distance to Centroid of Top System

Subdivided Area	Area A (mm^2)	Distance to Centroid from Reference Axis Y (mm)	A * Y (mm^3)
1 - Top Flange	$b_{f,top} * t_{f,top}$	$h - (t_{f,top} / 2)$	$[b_{f,top} * t_{f,top}] * [h - (t_{f,top} / 2)]$
2 - Web (above NA)	$t_w * (h - NA_{Y-Y} - t_{f,top})$	$NA_{Y-Y} + (h - NA_{Y-Y} - t_{f,top})/2$	$[NA_{Y-Y} * t_w * (h - NA_{Y-Y} - t_{f,top})] + [t_w * (h - NA_{Y-Y} - t_{f,top})^2 / 2]$
3 - Filleted Corner 1	$r^2 - 0.25\pi r^2$	$h - t_{f,top} - y_{corner2}$	$[r^2 - 0.25\pi r^2] * [h - t_{f,top} - y_{corner2}]$
4 - Filleted Corner 2	$r^2 - 0.25\pi r^2$	$h - t_{f,top} - y_{corner2}$	$[r^2 - 0.25\pi r^2] * [h - t_{f,top} - y_{corner2}]$
TOTALS	$\Sigma(A_{top})$	**TOTALS**	$\Sigma(A_{top} * y_{top})$

$Y_{TOP,SYSTEM} = \Sigma(A_{top} * y_{top}) / \Sigma(A_{top})$ ---> *units are Length, usually mm*

Distance to Centroid of Bottom System

Subdivided Area	Area A (mm^2)	Distance to Centroid from Reference Axis Y (mm)	A * Y (mm^3)
1 - Bottom Flange	$b_{f,bottom} * t_{f,bottom}$	$t_{f,bottom} / 2$	$[b_{f,bottom} * t_{f,bottom}] * [t_{f,bottom} / 2]$
2 - Web (below NA)	$t_w * (NA_{Y-Y} - t_{f,bottom})$	$t_{f,bottom} + (NA_{Y-Y} - t_{f,bottom})/2$	$[t_w * t_{f,bottom} * (NA_{Y-Y} - t_{f,bottom})] + [t_w * (NA_{Y-Y} - t_{f,bottom})^2 / 2]$
3 - Filleted Corner 3	$r^2 - 0.25\pi r^2$	$t_{f,bottom} + y_{corner2}$	$[r^2 - 0.25\pi r^2] * [t_{f,bottom} + y_{corner2}]$
4 - Filleted Corner 4	$r^2 - 0.25\pi r^2$	$t_{f,bottom} + y_{corner2}$	$[r^2 - 0.25\pi r^2] * [t_{f,bottom} + y_{corner2}]$
TOTALS	$\Sigma(A_{bottom})$	TOTALS	$\Sigma(A_{bottom} * y_{bottom})$

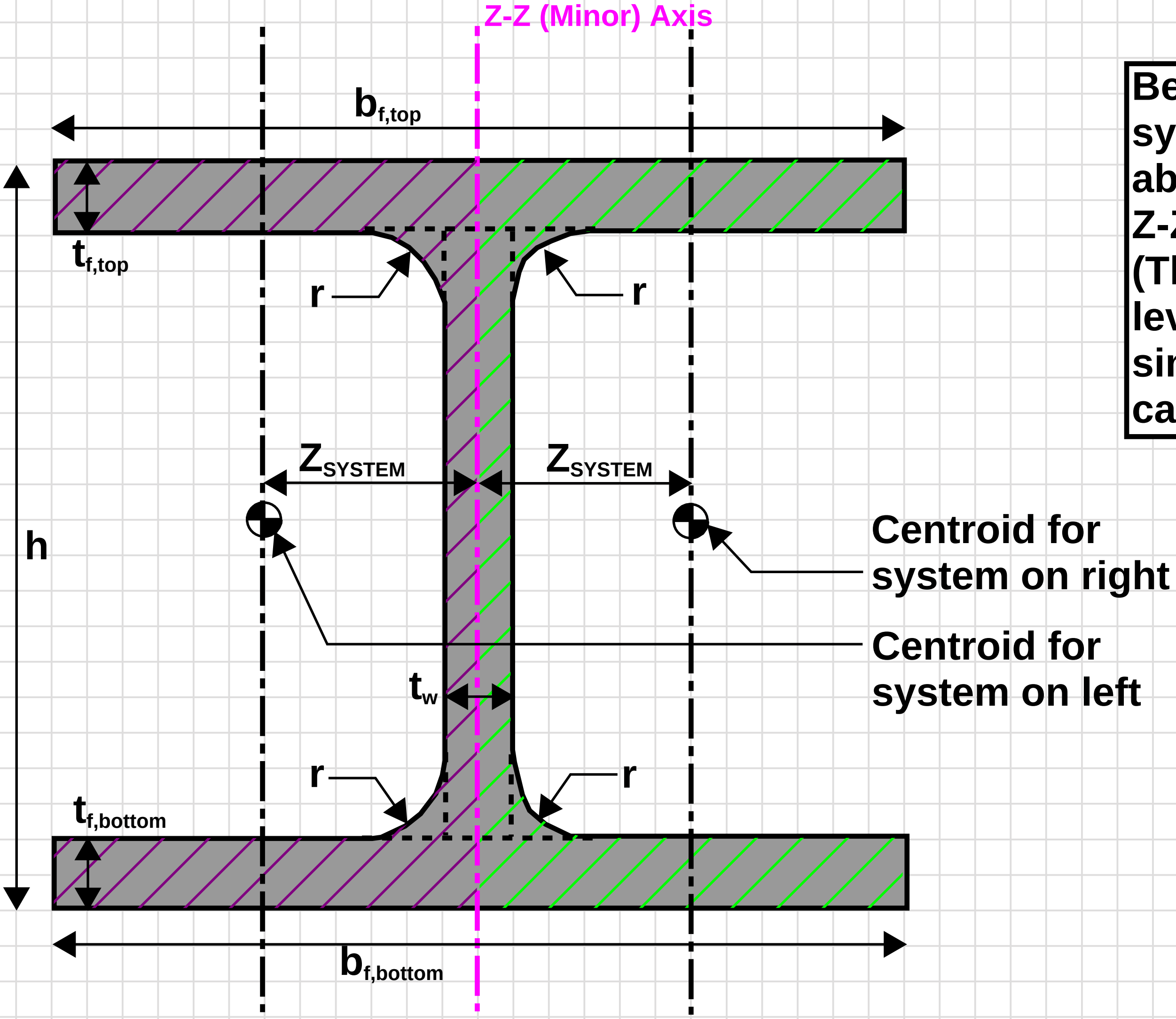
$$Y_{BOTTOM,SYSTEM} = \Sigma(A_{bottom} * y_{bottom}) / \Sigma(A_{bottom})$$ *---> units are Length, usually mm*

Calculate Plastic Section Modulus About Major Y-Y Axis

$$W_{pl,y} = [(Y_{TOP,SYSTEM} - NA_{Y-Y}) * \Sigma(A_{top})] + [(NA_{Y-Y} - Y_{BOTTOM,SYSTEM}) * \Sigma(A_{bottom})]$$

units are Length^3, usually mm^3

Plastic Section Modulus About Minor Z-Z Axis



Beam is symmetric about the Z-Z axis (This is leveraged to simplify the calculation)

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Distance to Centroid from Z-Z Axis

Subdivided Area	Area A (mm^2)	Distance to Centroid from Z Axis (mm)	A * Z (mm^3)
1 - Top Flange	$b_{f,top}$ * $t_{f,top}$ * 1/2	$b_{f,top}$ / 4	$(b_{f,top})^2$ * $t_{f,top}$ * 1/8
2 - Bottom Flange	$b_{f,bottom}$ * $t_{f,bottom}$ * 1/2	$b_{f,bottom}$ / 4	$(b_{f,bottom})^2$ * $t_{f,bottom}$ * 1/8
3 - Web	t_w * (h - $t_{f,top}$ - $t_{f,bottom}$) * 1/2	t_w / 4	$(t_w)^2$ * (h - $t_{f,top}$ - $t_{f,bottom}$) * (1/8)
4 - Filleted Corner 2	$r^2 - 0.25\pi r^2$	$(t_w / 2) + y_{corner2}$	$[r^2 - 0.25\pi r^2] * [(t_w / 2) + y_{corner2}]$
5 - Filleted Corner 4	$r^2 - 0.25\pi r^2$	$(t_w / 2) + y_{corner2}$	$[r^2 - 0.25\pi r^2] * [(t_w / 2) + y_{corner2}]$
TOTALS	$\Sigma(A_{zz})$	TOTALS	$\Sigma(A_{zz} * Z)$

$$Z_{SYSTEM} = \Sigma(A_{zz} * Z) / \Sigma(A_{zz})$$ ---> *units are Length, usually mm*

Calculate Plastic Section Modulus About Minor Z-Z Axis

$$W_{pl,z} = 2 * [\ Z_{SYSTEM} * \Sigma(A_{zz})\ ]$$ ---> *units are Length³, usually mm³*

Calculate Radius of Gyration

Radius of Gyration about Major Y-Y Axis

$$i_y = \sqrt{\frac{I_{Y-Y}}{\sum(A)}}$$

Second moment of area about major Y-Y axis

Cross sectional area of entire cross section

Radius of Gyration about Minor Z-Z Axis

$$i_z = \sqrt{\frac{I_{Z-Z}}{\sum(A)}}$$

Second moment of area about minor Z-Z axis

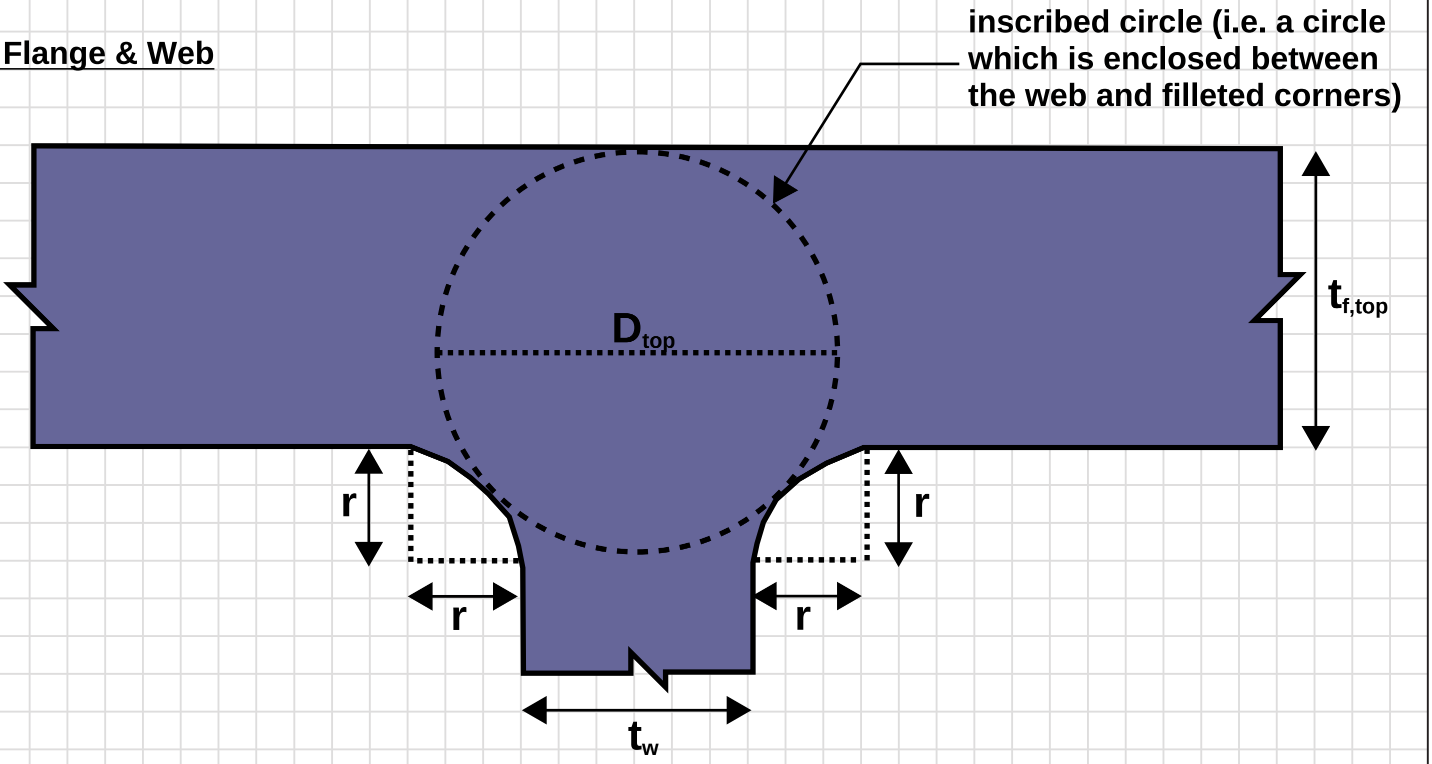
Cross sectional area of entire cross section

units for radius of gyration are Length, usually mm

Calculate Torsion Constant Inertia

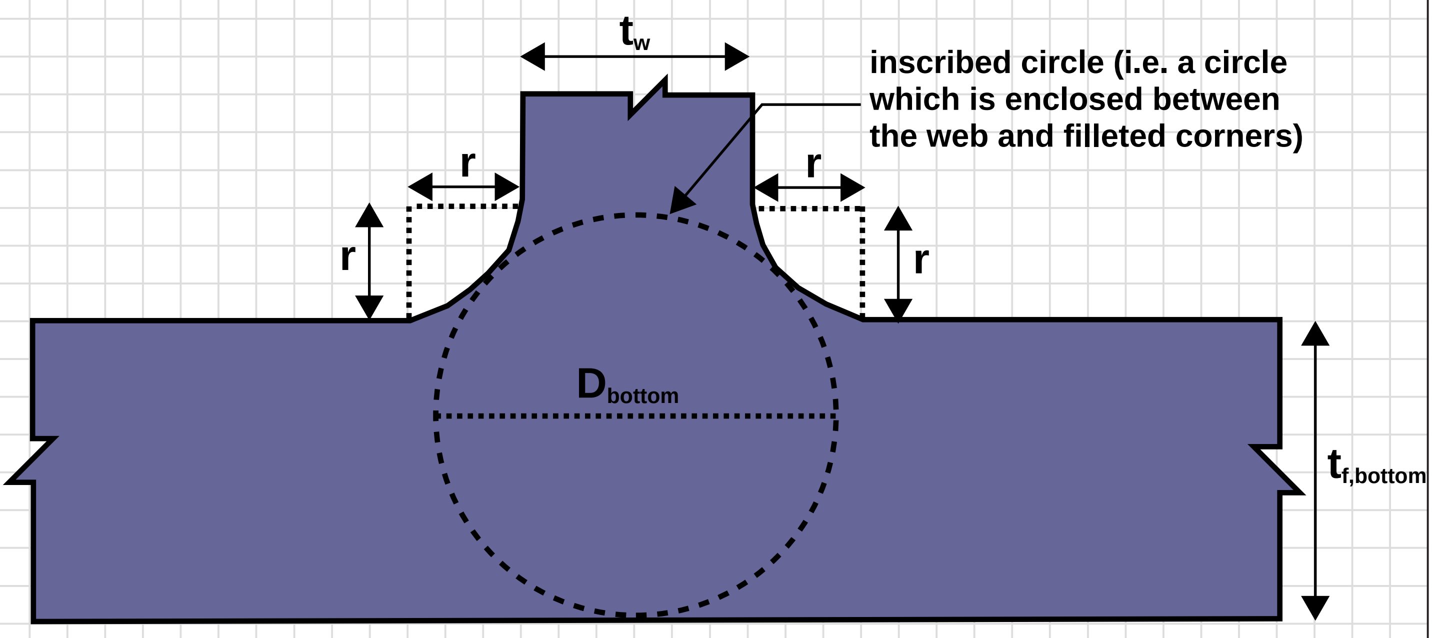
Inscribed Circle at Web-Flange Junction

Top Flange & Web



Diameter of inscribed circle $D_{top} = \dfrac{(t_{f,top} + r)^2 + t_w(r + \frac{t_w}{4})}{(2r + t_{f,top})}$

Bottom Flange & Web



Diameter of inscribed circle $D_{bottom} = \dfrac{(t_{f,bottom} + r)^2 + t_w(r + \frac{t_w}{4})}{(2r + t_{f,bottom})}$

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Empirical Alpha Correction Factor for Filleted Corners

Top Flange & Web Fillets

$$\alpha_{top} = -0.042 + 0.2204\frac{t_w}{t_{f,top}} + 0.1355\frac{r}{t_{f,top}} - 0.0865\frac{t_w r}{t_{f,top}^2} - 0.0725\frac{t_w^2}{t_{f,top}^2}$$

Bottom Flange & Web Fillets

$$\alpha_{bottom} = -0.042+0.2204\frac{t_w}{t_{f,bottom}}+0.1355\frac{r}{t_{f,bottom}}-0.0865\frac{t_w r}{t_{f,bottom}^2}-0.0725\frac{t_w^2}{t_{f,bottom}^2}$$

Torsion Constant

$I_{t,top,flange} = \frac{b_{f,top} * t_{f,top}^3}{3}$	
$I_{t,bottom,flange} = \frac{b_{f,bottom} * t_{f,bottom}^3}{3}$	
$I_{t,web} = \frac{(h - t_{f,top} - t_{f,bottom}) * t_w^3}{3}$	
$I_{t,top,circle} = \alpha_{top} * D_{top}^4$	
$I_{t,bottom,circle} = \alpha_{bottom} * D_{bottom}^4$	
$I_{t,top,correction} = -2 * 0.105 * t_{t,top}^4$	
$I_{t,bottom,correction} = -2 * 0.105 * t_{t,bottom}^4$	
TOTAL	**I_t = sum of items above**

Notes:

The torsion constant is expressed in units of length^4. The symbol I_t is used in Eurocode however it can sometimes be referred to a J in older codes of practise

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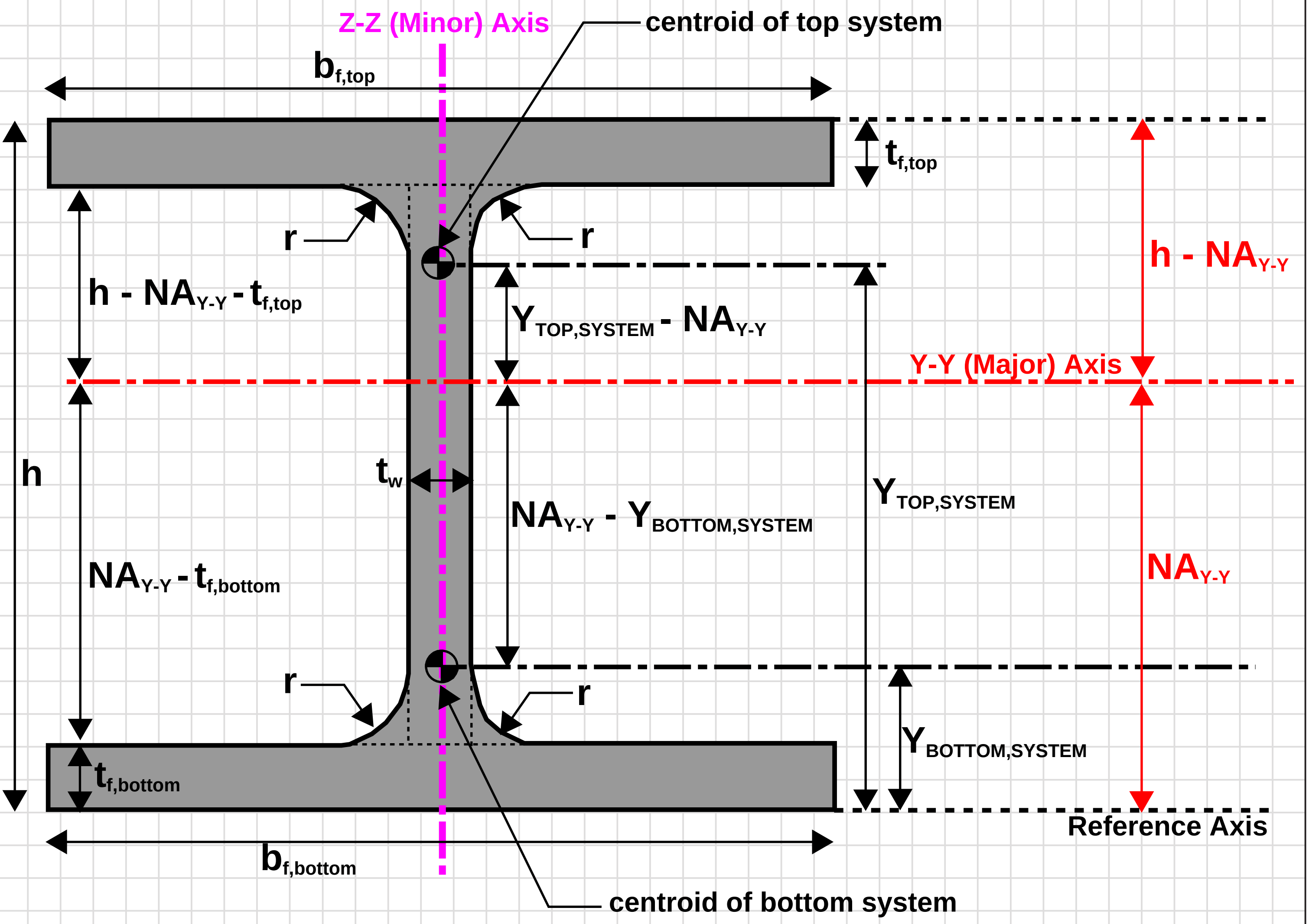
Calculate Shear Centre



The centroids of the top and bottom systems of the beam were calculated previously where we worked out the plastic section modulus.

Second Moment of Area of "Top System" about Z-Z Axis

Subdivided Area	Second Moment of Area about Centroid of Subdivided Element I_z (mm^4)	Distance from Neutral Axis to Centroid of Subdivided Element Z_{bar} (mm)	(Z_{bar} for each element)^2 * (Area of each element) Z_{bar}^2 * A
1 - Top Flange	$t_{f,top}$ * $b_{f,top}^3$ * (1/12)	0	(Z_{bar}^2) * $t_{f,top}$ * $b_{f,top}$
2 - Web Above NA	(h - NA_{Y-Y} - $t_{f,top}$) * t_w^3 * (1/12)	0	(Z_{bar}^2) * t_w * (h - NA_{Y-Y} - $t_{f,top}$)
3 - Filleted Corner 1	$I_{corner} = 4r^4(\frac{1}{12} - \frac{\pi}{64}) - y_{corner1}^2(r^2 - \frac{\pi r^2}{4})$	-(t_w / 2) - $y_{corner2}$	$(Z_{bar})^2$ * (r^2 - 0.25πr^2)
4 - Filleted Corner 2	$I_{corner} = 4r^4(\frac{1}{12} - \frac{\pi}{64}) - y_{corner1}^2(r^2 - \frac{\pi r^2}{4})$	(t_w / 2) + $y_{corner2}$	$(Z_{bar})^2$ * (r^2 - 0.25πr^2)
TOTALS	$\Sigma(I_Z)$	**TOTALS**	$\Sigma(Z_{bar}$ * A)

$$I_{f,top,system} = \Sigma(I_Z) + \Sigma(Z_{bar} * A)$$

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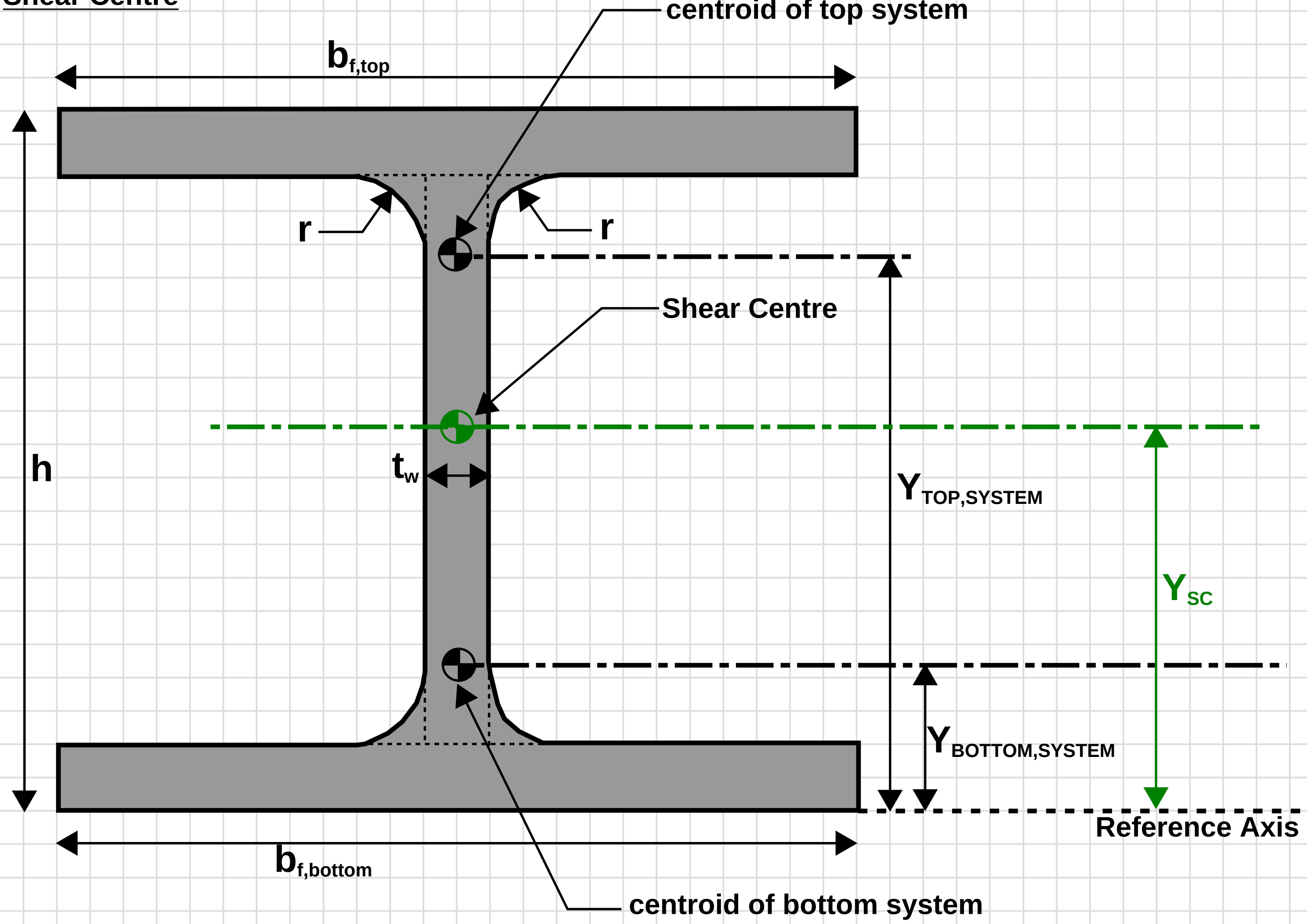
Second Moment of Area of "Bottom System" about Z-Z Axis

Subdivided Area	Second Moment of Area about Centroid of Subdivided Element I_z (mm^4)	Distance from Neutral Axis to Centroid of Subdivided Element Z_{bar} (mm)	(Z_{bar} for each element)^2 * (Area of each element) $Z_{bar}^2 * A$
1 - Bottom Flange	$t_{f,bottom} * b_{f,bottom}^3 * (1/12)$	0	$(Z_{bar}^2) * t_{f,bottom} * b_{f,bottom}$
2 - Web Below NA	$(NA_{Y-Y} - t_{f,bottom}) * t_w^3 * (1/12)$	0	$(Z_{bar}^2) * t_w * (NA_{Y-Y} - t_{f,bottom})$
3 - Filleted Corner 3	$I_{corner} = 4r^4(\frac{1}{12} - \frac{\pi}{64}) - y_{corner1}^2(r^2 - \frac{\pi r^2}{4})$	$-(t_w / 2) - y_{corner2}$	$(Z_{bar})^2 * (r^2 - 0.25\pi r^2)$
4 - Filleted Corner 4	$I_{corner} = 4r^4(\frac{1}{12} - \frac{\pi}{64}) - y_{corner1}^2(r^2 - \frac{\pi r^2}{4})$	$(t_w / 2) + y_{corner2}$	$(Z_{bar})^2 * (r^2 - 0.25\pi r^2)$
TOTALS	$\Sigma(I_Z)$	**TOTALS**	$\Sigma(Z_{bar} * A)$

$$I_{f,bottom,system} = \Sigma(I_Z) + \Sigma(Z_{bar} * A)$$

Shear Centre



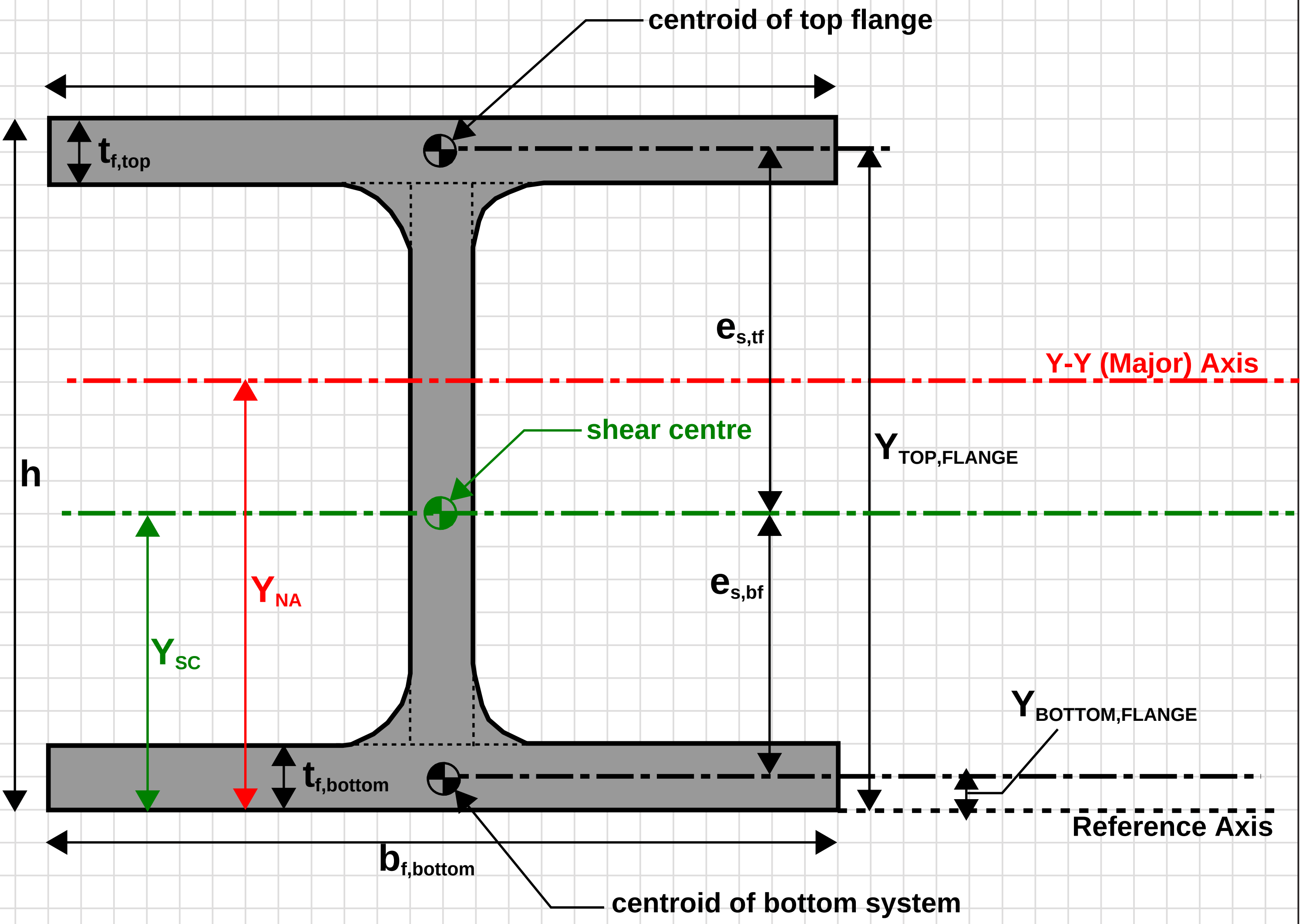
$$Y_{SC} = \frac{(Y_{TOP,SYSTEM} - Y_{BOTTOM,SYSTEM}) * I_{f,top}}{(I_{f,top} + I_{f,bottom})}$$

Calculate Warping Constant

Method 1: Consider Only the Beam Flanges

This method is the one recommended by the Steel Construction Institute (SCI) in document P385: Design of Steel Beams in Torsion - Annex B.

The method involves only considering the flanges of the beam to be important when calculating the warping constant I_w. This is incidentally the same method used to find the warping constant for all the Universal beams and Universal columns provided in the Blue Book (also provided by the SCI).



The distances to the centroids for the top and bottom system shown in the diagram above is different from that shown previously when working out the shear centre. We only care about the flanges.

Distances to Centroids of Top and Bottom Flanges

$$Y_{TOP,FLANGE} = h - t_{f,top} / 2$$

$$Y_{BOTTOM,FLANGE} = t_{f,bottom} / 2$$

Find Second Moments of Area of Top and Bottom Flanges about Minor Z-Z Axis

Second moment of area of top flange about minor axis

$$I_{f,top} = \frac{t_{f,top} * b_{f,top}^2}{12}$$

Second moment of area of bottom flange about minor axis

$$I_{f,bottom} = \frac{t_{f,bottom} * b_{f,bottom}^2}{12}$$

Find Warping Constant of Top System

$$e_{s,tf} = Y_{TOP,FLANGE} - Y_{SC}$$

$$I_{w,top} = I_{f,top} * e_{s,tf} * (Y_{TOP,FLANGE} - Y_{BOTTOM,FLANGE})$$

Find Warping Constant of Bottom System

$$e_{s,bf} = Y_{SC} - Y_{BOTTOM,FLANGE}$$

$$I_{w,bottom} = I_{f,bottom} * e_{s,bf} * (Y_{TOP,FLANGE} - Y_{BOTTOM,FLANGE})$$

Final Warping Constant

$$I_w = \min(I_{w,top} ; I_{w,bottom})$$

Method 2: Consider Entire Section As Contributing

This assessment provides better warping constant results which are not as conservative and not as realistic.

The same steps as method 1 are taken but the stiffness of the beams web and filleted corners are also taken into account.

For more information on how to calculate the second moment of area about the minor axis refer to the previous formulas within this document.

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